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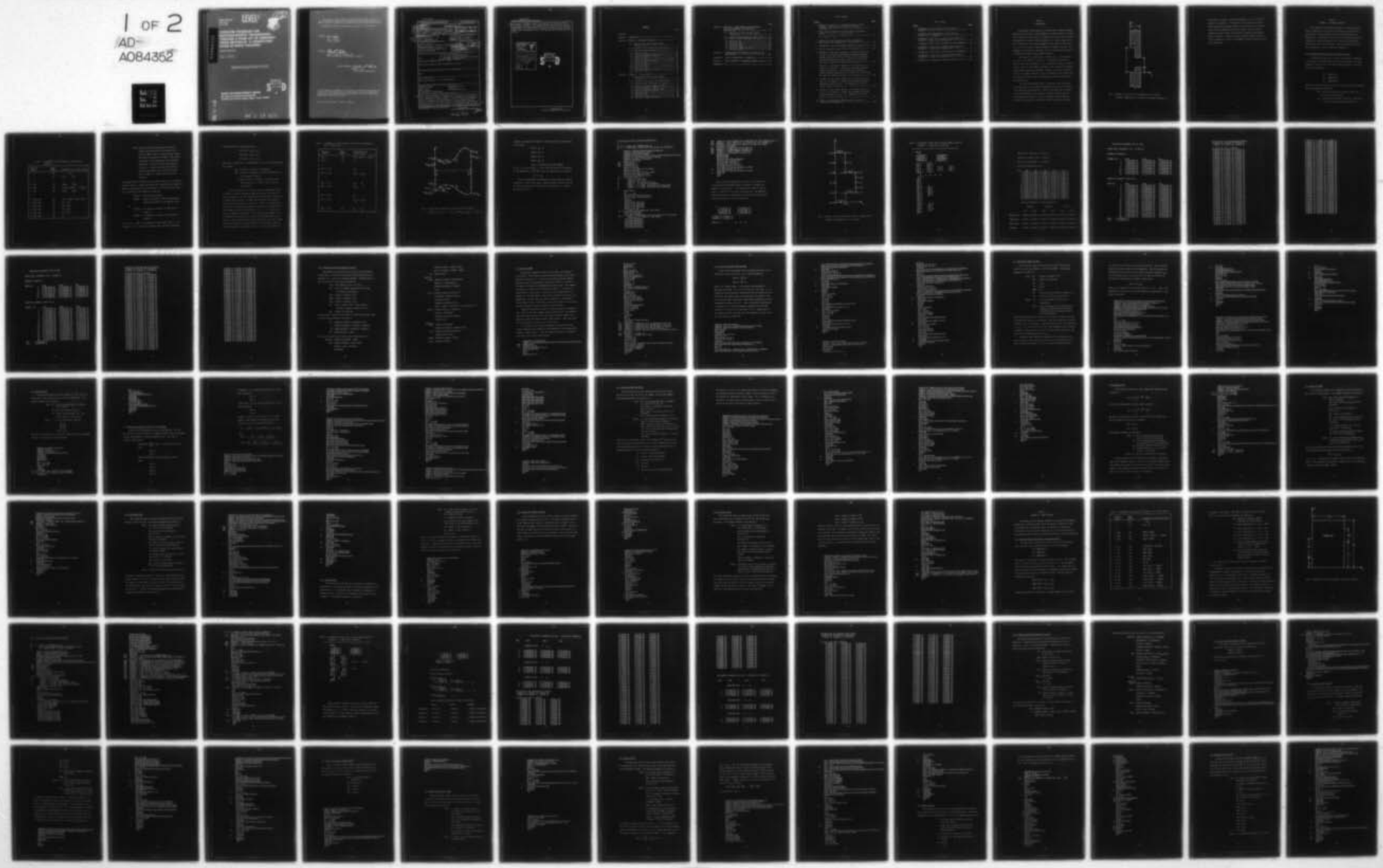

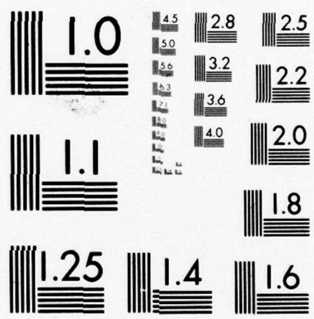

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RADC-TR-80-59
Phase Report
March 1980

COMPUTER PROGRAMS FOR ELECTROMAGNETIC TRANSMISSION THROUGH A FILLED SLIT OF ARBITRARY CROSS SECTION IN A CONDUCTING PLANE OF FINITE THICKNESS

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David T. Auckland
Roger F. Harringtor

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, New York 13441

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

Three computer programs are presented for the analysis of electromagnetic transmission through a filled slit of arbitrary cross section in a conducting plane of finite thickness. The first and second programs utilize a non-modal and modal formulation, respectively, which are derived in detail in an earlier report. Some approximate solutions are available when the slit cross section is a rectangular shape and these are investigated in the third program. The programs are all written

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in Fortran IV language. Quantities computed are equivalent electric and magnetic currents on the contour defining the slit cross section, transmission coefficient, gain patterns, and normalized field patterns. Sample input and output data are presented for each program. Detailed instructions concerning the use of each program and its capabilities are given.

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Chapter 1

INTRODUCTION

The purpose of this report is to document three computer programs written for the analysis of electromagnetic transmission through a filled slit of arbitrary cross section in a conducting plane of finite thickness. The formulation is given in detail elsewhere [1]. Several equations in [1] are referred to here and are numbered as they appear in that report (i.e., Eq. (3-2) refers to equation number 2 of Chapter 3 of [1]). The language of the programs is Fortran IV and all computations were done on an IBM 370-155 computer using a WATFIV compiler.

An example for a typical slit is shown in Fig. 1, where regions a, b, and c are taken to be free space. The dimensions are given in terms of free space wavelength, λ_o. For illustrative purposes, computations are done for this example in Chapters 2 and 3, where the non-modal and the modal programs are discussed. Chapter 4 deals with a specialization of the modal solution to a slit of rectangular cross section, and the program described is used to investigate approximate solutions for this special case.

The slit cross section of Fig. 1 is special in the sense that either the modal or the non-modal formulation may be used to solve for its transmission characteristics. Appendix A discusses how to compare the results obtained from the computer programs using these two different methods of solution. Since the non-modal solution comprises the bulk of reference [1], the Fortran variable names which are used

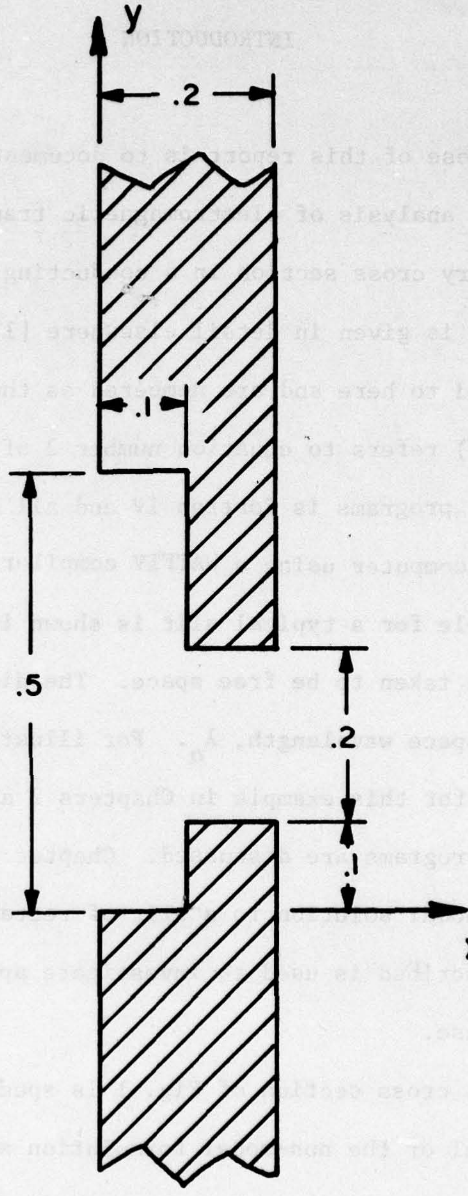

Fig. 1. Example of a slit cut in a conducting screen of finite thickness. Dimensions are in terms of free space wavelength λ_o.

consistently in Program I and have counterparts in [1] are listed for reference in Appendix B. A solution of a slit problem by the modal method requires only magnetic currents to be solved for over portions of the contour defining the slit cross section. Hence, to obtain tangential magnetic fields on this contour, additional computational effort is required. Formulas for these tangential magnetic fields are given in Appendix C in terms of the magnetic currents which were solved for in Program II.

Chapter 2

PROGRAM I - NON-MODAL SOLUTION

The purpose of this computer program is to analyze the transmission properties of a filled slit of arbitrary cross section in a ground plane of finite thickness by the non-modal formulation [1, Chapters 1-4]. Descriptions of how to use the program and set up the required data cards are presented first, followed by detailed descriptions of all of the subroutines. Fortran variable names, which have counterparts in [1], are defined in Appendix B.

2.1.1 Required Input Data and Main Program Description

The input data required by the user is read from two sequences of data cards in the main program and in subroutine CDATA. The first sequence of cards is read in the main program according to the following format statements:

```
100   FORMAT(6I5)
101   FORMAT(2E20.7)
102   FORMAT(6E11.4)
```

The data in the first sequence of cards is arranged according to Table 1. The data variables are defined by:

NGQ = order of the Gaussian quadrature formula used for numerical integration.

A,T = arrays containing weights and nodes, respectively, of Gaussian quadrature formula divided by 2.

Table 1. Arrangement of first sequence of data cards for Program I.

Data Card Number	Format Number	Information to be Typed on Card	
1	101	NGQ	
2	101	A(1)	T(1)
.	.	.	.
.	.	.	.
.	.	.	.
1 + NGQ	101	A(NGQ)	T(NGQ)
2 + NGQ	100	IJOB(1), IJOB(2),..., IJOB(6)	
3 + NGQ	102	PHI(1)	PHI(2)
.	.	.	.
.	.	.	.
.	.	.	.
1 + NEX + NGQ	102	PHI(2 * NEX-1), PHI(2 * NEX)	
2 + NEX + NGQ	102	MUA , EPSA	
3 + NEX + NGQ	102	MUB , EPSB	
4 + NEX + NGQ	102	MUC , EPSC	
5 + NEX + NGQ	102	FMC , DPHI	
6 + NEX + NGQ	100	NI	

IJOB = Integer option array allowing user to specify various options available in the program as defined by comment cards in the main program listing. The number of excitations is IJOB(3) = NEX, either plane waves or line sources, all to be considered separately. If any of the other IJOB values are not equal to 1, then that particular task indicated by the comment cards will not be executed. Also if IJOB(6) = 1, IJOB(5) must also be set equal to 1.

The program obtains a solution for an incident plane wave or line source placed in region a. Several excitations may be specified for a given slit problem, each of which is solved separately. For NEX excitations, each one is specified by two locations in array PHI:

PHI(2k-1) = 0, plane wave excitation.

PHI(2k) = angle of incidence of plane wave measured in degrees with respect to the negative x axis.

or

PHI(2k-1) = x coordinate, in meters, of line source in region a.

PHI(2k) = y coordinate, in meters, of line source in region a.

for k = 1,2,..., NEX. The coordinates used are those of Fig. 1. All regions a, b, or c may be lossy and hence their electrical properties

are specified by the complex quantities:

$$\text{MUA, EPSA} = \mu_a/\mu_o,\ \varepsilon_a/\varepsilon_o$$

$$\text{MUB, EPSB} = \mu_b/\mu_o,\ \varepsilon_b/\varepsilon_o$$

$$\text{MUC, EPSC} = \mu_c/\mu_o,\ \varepsilon_c/\varepsilon_o$$

where each is specified by a complex number x-jy for x and y non-negative real numbers.

FMC = Frequency of excitation in Megahertz.

DPHI = Increment, in degrees, at which field patterns are to be computed in region c.

NI = Number of points in region c at which far field computations are to be made in order to obtain field patterns.

The second sequence of data cards, which are arranged according to Table 2, are read in subroutine CDATA and are used to specify the slit cross section. The data variables are defined in Fig. 2, where the coordinate frame is shown. The number NS1 equals the total number of cards in the first sequence of data cards. In Fig. 2, the endpoints of the lines Γ_i are sequentially numbered clockwise from the origin by the node numbers N_i stored in array ND for i = 1, 2, ..., 5 so that each line Γ_i is broken up into ND(i+1) - ND(i) straight line segments. The x coordinates of the points on Γ_1 and Γ_3 are not needed except at the ends of Γ_2 and Γ_4 because these lines are parallel to the y axis. There are N_5+1 data cards in the second sequence. As this data is read in, the vector quantities appearing in Fig. 5 of [1] associated with each

Table 2. Arrangement of second sequence of data cards for Program I, NS1 = 6 + NEX + NGQ.

Data Card Number	Format Number	Information to be Typed on Cards
NS1 + 1	100	ND(1), ND(2),..., ND(5)
NS1 + 2	102	Y_1
.	.	.
.	.	.
.	.	.
NS1 + 1 + N_2		Y_{N_2}
NS1 + 2 + N_2		X_{N_2+1}, Y_{N_2+1}
.		. .
		. .
		. .
NS1 + 1 + N_3		X_{N_3}, Y_{N_3}
NS1 + 2 + N_3		Y_{N_3+1}
.		.
.		.
NS1 + 1 + N_4		Y_{N_4}
NS1 + 2 + N_4		X_{N_4+1}, Y_{N_4+1}
.		. .
.		. .
NS1 + 1 + N_5	102	X_{N_5}, Y_{N_5}

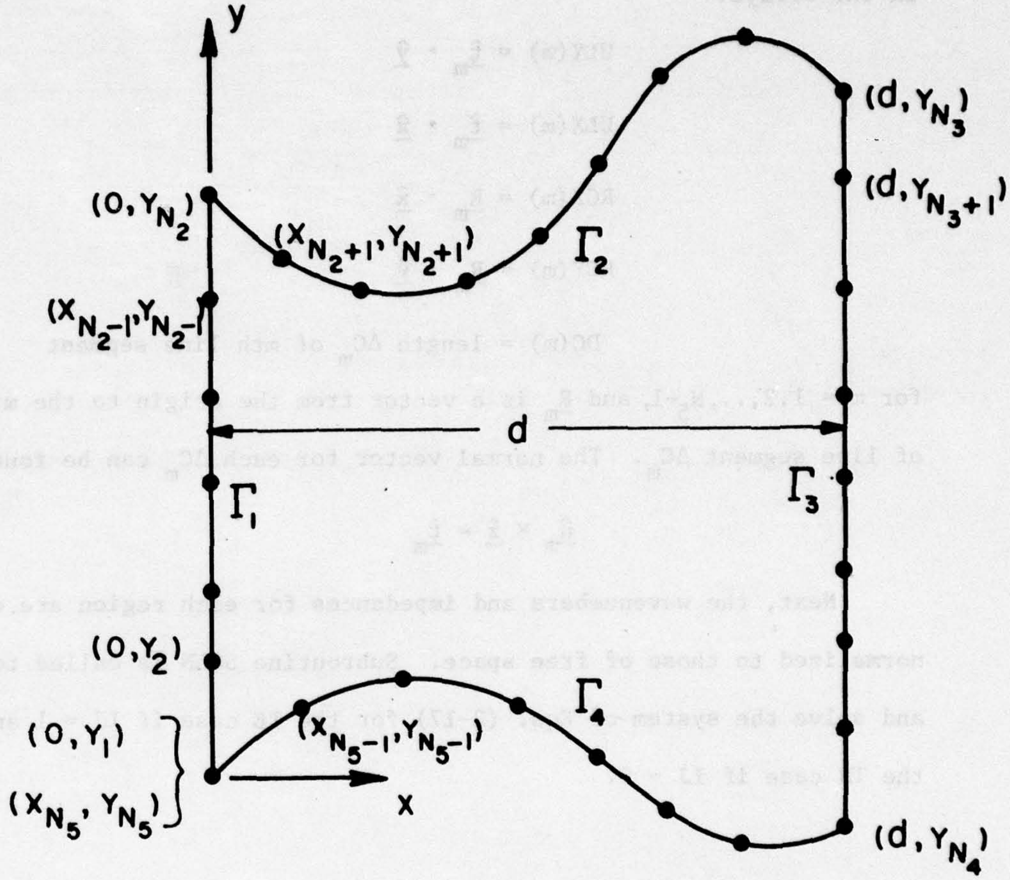

Fig. 2. Points used as input data to specify slit cross section contour C, $x_5 = x_1 = \ldots = x_{N_2} = 0$ and $x_{N_3} = x_{N_3+1} = \ldots x_{N_4} = d$.

straight line segment are computed. These are stored in common blocks in the arrays:

$$\text{ULY}(m) = \underline{\hat{t}}_m \cdot \underline{\hat{y}}$$

$$\text{ULX}(m) = \underline{\hat{t}}_m \cdot \underline{\hat{x}}$$

$$\text{RCX}(m) = \underline{R}_m \cdot \underline{\hat{x}}$$

$$\text{RCY}(m) = \underline{R}_m \cdot \underline{\hat{y}}$$

$$\text{DC}(m) = \text{length } \Delta C_m \text{ of mth line segment}$$

for m = 1,2,..,N_5-1, and $\underline{R}_m$ is a vector from the origin to the midpoint of line segment ΔC_m. The normal vector for each ΔC_m can be found from

$$\underline{\hat{n}}_m \times \underline{\hat{z}} = \underline{\hat{t}}_m$$

Next, the wavenumbers and impedances for each region are computed normalized to those of free space. Subroutine SOLN is called to set up and solve the system of Eqs. (2-17) for the TE case if IJ = 1 and for the TM case if IJ = 2.

2.1.2 Listing of Main Program and Sample Output

```
C
C---------THICK SLIT PROGRAM ONE...
C----------NON-MODAL FORMULATION FOR THICK SLIT PROBLEM...
C
      COMPLEX KA,KB,KC,ETA,ETB,ETC,MUA,MUB,MUC
      COMPLEX EPSA,EPSB,EPSC,CSQRT
      DIMENSION PHI(100),IJOB(6)
      COMMON /CUV/ULX(60),ULY(60),ND(5)/C/RCX(60),RCY(60),DC(60)
      COMMON /B/ ETA,ETB,ETC,KA,KB,KC,WAVNO
      COMMON /GQI/A(10),T(10),NGQ
      DATA PI/3.141593/
100   FORMAT(6I5)
101   FORMAT(2E20.7)
102   FORMAT(6E11.4)
      READ(1,100) NGQ
      READ(1,101)(A(I),T(I),I=1,NGQ)
      WRITE(3,100) NGQ
      WRITE(3,101)(A(I),T(I),I=1,NGQ)
      READ(1,100)(IJOB(I),I=1,6)
      WRITE(3,100)(IJOB(I),I=1,6)
C---------OPTIONS ARE...
C          IJOB(1)=1...TE CASE
C          IJOB(2)=1...TM CASE
C          IJOB(3) = NO. OF EXCITATIONS
C          IJOB(4) = 1...PRINT OUT CURRENTS
C          IJOB(5) = 1...COMP. TRANSMISSION COEFFICIENT.
C          IJOB(6) = 1...COMP. GAIN AND FIELD PATTERNS
C
      NEX=IJOB(3)
      K=1
      DO 1 I=1,NEX
      READ(1,102) PHI(K),PHI(K+1)
      WRITE(3,102) PHI(K),PHI(K+1)
      K=K+2
1     CONTINUE
      READ(1,102) MUA,EPSA
      READ(1,102) MUB,EPSB
      READ(1,102) MUC,EPSC
      READ(1,102) FMC,DPHI
      READ(1,100) NI
C---------COMPUTE WAVENUMBER OF FREE SPACE
      WAVNO=PI*FMC/150.
      CALL CDATA(WAVNO)
C---------COMPUTE WAVENUMBERS AND IMPEDANCES OF ALL REGIONS
C          NORMALIZED BY THOSE OF FREE SPACE
      KA=CSQRT(MUA*EPSA)
      ETA=CSQRT(MUA/EPSA)
      KB=CSQRT(MUB*EPSB)
      ETB=CSQRT(MUB/EPSB)
      KC=CSQRT(MUC*EPSC)
      ETC=CSQRT(MUC/EPSC)
```

```
200     FORMAT('1',20X,'NORMALIZED PARAMETERS FOR EACH REGION ARE...')
201     FORMAT('-',27X,'REGION A',14X,'REGION B',14X,'REGION C')
202     FORMAT('-',23X,'REAL',7X,'IMAG',7X,'REAL',7X,'IMAG',
       17X,'REAL',7X,'IMAG')
203     FORMAT('-','PERMITTIVITY',8X,6E11.4)
204     FORMAT('-','PERMEABILITY',8X,6E11.4)
205     FORMAT('-','WAVE NUMBER',9X,6E11.4)
206     FORMAT('-','IMPEDANCE',11X,6E11.4)
        WRITE(3,200)
        WRITE(3,201)
        WRITE(3,202)
        WRITE(3,203) EPSA,EPSB,EPSC
        WRITE(3,204) MUA,MUB,MUC
        WRITE(3,205) KA,KB,KC
        WRITE(3,206) ETA,ETB,ETC
        IF(IJOB(1).NE.1) GO TO 10
        CALL SOLN(1,IJOB,PHI,ND,NEX,NI,CPHI)
10      IF(IJOB(2).NE.1) GO TO 11
        CALL SOLN(2,IJOB,PHI,ND,NEX,NI,CPHI)
11      CONTINUE
        STOP
        END
```

Sample output is presented for the slit in Fig. 1 where its cross section has been approximated by a number of straight line segments as shown in Fig. 3. The required data cards are shown in Table 3. Only the $\phi^i = 0$ results are printed out. The quantities PTA and PTC printed out right after the currents are the real power flow across apertures Γ_1 and Γ_3 respectively.

```
    4
       0.1739274E 00        -0.4305682E 00
       0.1739274E 00         0.4305682E 00
       0.3260725E 00        -0.1699905E 00
       0.3260725E 00         0.1699905E 00
    1     1     2     1     1     1
 0.0000E 00 0.0000E 00
 0.0000E 00 0.4500E 02

 ARRAY ND =     1     6    12    16    21
```

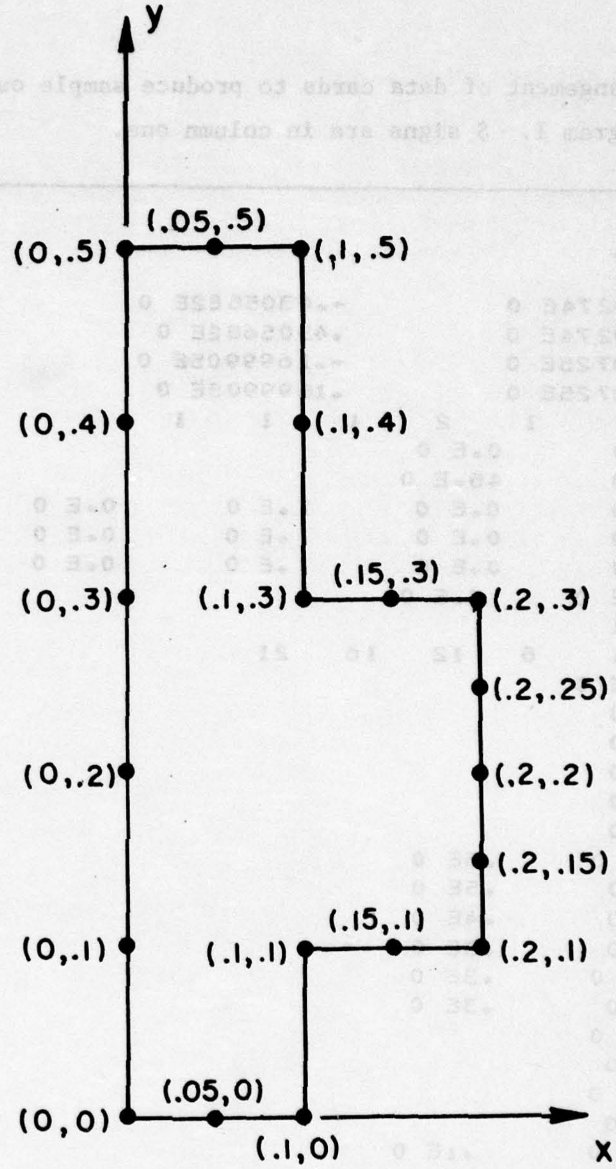

Fig. 3. Contour C of slit cross section in Fig. 1 broken up into straight line segments, ND = 1,6,12,16,21.

Table 3. Arrangement of data cards to produce sample output in Program I. $ signs are in column one.

```
$DATA
   4
.1739274E 0          -.4305682E 0
.1739274E 0          .4305682E 0
.3260725E 0          -.1699905E 0
.3260725E 0          .1699905E 0
   1     1    2    1    1    1
0.E 0      0.E 0
0.E 0      45.E 0
1.E 0      0.E 0      1.E 0      0.E 0
1.E 0      0.E 0      1.E 0      0.E 0
1.E 0      0.E 0      1.E 0      0.E 0
300.E 0    2.E 0
   91
   1     6    12    16    21
  0.E 0
.1E 0
.2E 0
.3E 0
.4E 0
.5E 0
.05E 0     .5E 0
.1E 0      .5E 0
.1E 0      .4E 0
.1E 0      .3E 0
.15E 0     .3E 0
.2E 0      .3E 0
.25E 0
.2E 0
.15E 0
.1E 0
.15E 0      .1E 0
.1E 0      .1E 0
.1E 0      0.E 0
.05E 0      0.E 0
0.E 0      0.E 0
$STOP
//
```

WIDTH OF SLIT IN REGION A= 0.5000E 00

THICKNESS OF GROUND PLANE = 0.2000E 00

WIDTH OF SLIT IN REGION C = 0.2000E 00

INPUT DATA DEFINING CONTOUR...

NODE	ULX	ULY	RCX	RCY	DC
1	0.0000E 00	0.1000E 01	0.0000E 00	0.5000E-01	0.1000E 00
2	0.0000E 00	0.1000E 01	0.0000E 00	0.1500E 00	0.1000E 00
3	0.0000E 00	0.1000E 01	0.0000E 00	0.2500E 00	0.1000E 00
4	0.0000E 00	0.1000E 01	0.0000E 00	0.3500E 00	0.1000E 00
5	0.0000E 00	0.1000E 01	0.0000E 00	0.4500E 00	0.1000E 00
6	0.1000E 01	0.0000E 00	0.2500E-01	0.5000E 00	0.5000E-01
7	0.1000E 01	0.0000E 00	0.7500E-01	0.5000E 00	0.5000E-01
8	0.0000E 00	-0.1000E 01	0.1000E 00	0.4500E 00	0.1000E 00
9	0.0000E 00	-0.1000E 01	0.1000E 00	0.3500E 00	0.1000E 00
10	0.1000E 01	0.0000E 00	0.1250E 00	0.3000E 00	0.5000E-01
11	0.1000E 01	0.0000E 00	0.1750E 00	0.3000E 00	0.5000E-01
12	0.0000E 00	-0.1000E 01	0.2000E 00	0.2750E 00	0.5000E-01
13	0.0000E 00	-0.1000E 01	0.2000E 00	0.2250E 00	0.5000E-01
14	0.0000E 00	-0.1000E 01	0.2000E 00	0.1750E 00	0.5000E-01
15	0.0000E 00	-0.1000E 01	0.2000E 00	0.1250E 00	0.5000E-01
16	-0.1000E 01	0.0000E 00	0.1750E 00	0.1000E 00	0.5000E-01
17	-0.1000E 01	0.0000E 00	0.1250E 00	0.1000E 00	0.5000E-01
18	0.0000E 00	-0.1000E 01	0.1000E 00	0.5000E-01	0.1000E 00
19	-0.1000E 01	0.0000E 00	0.7500E-01	0.0000E 00	0.5000E-01
20	-0.1000E 01	0.0000E 00	0.2500E-01	0.0000E 00	0.5000E-01

NORMALIZED PARAMETERS FOR EACH REGION ARE...

	REGION A		REGION B		REGION C	
	REAL	IMAG	REAL	IMAG	REAL	IMAG
PERMITTIVITY	0.1000E 01	0.0000E 00	0.1000E 01	0.0000E 00	0.1000E 01	0.0000E 00
PERMEABILITY	0.1000E 01	0.0000E 00	0.1000E 01	0.0000E 00	0.1000E 01	0.0000E 00
WAVE NUMBER	0.1000E 01	0.0000E 00	0.1000E 01	0.0000E 00	0.1000E 01	0.0000E 00
IMPEDANCE	0.1000E 01	0.0000E 00	0.1000E 01	0.0000E 00	0.1000E 01	0.0000E 00

EQUIVALENT CURRENTS FOR TE CASE

PLANE WAVE INCIDENCE, PHI = 0.000E 00

MAGNETIC CURRENTS...

SEGMENT NO.	REAL	IMAG.	MAG.
1	0.1024279E 01	0.5102060E 00	0.1144315E 01
2	0.5400261E 00	0.4334849E 00	0.6924863E 00
3	0.6626759E 00	0.4761689E 00	0.8160123E 00
4	0.4607493E 00	0.6667711E 00	0.8104773E 00
5	0.8722345E 00	0.8758489E 00	0.1236084E 01
12	0.6956310E 00	0.1829186E 01	0.1956993E 01
13	0.3171800E 00	0.1155327E 01	0.1198074E 01
14	0.3019215E 00	0.1141146E 01	0.1180410E 01
15	0.6531895E 00	0.1776711E 01	0.1892975E 01

ELECTRIC CURRENTS TIMES ETA...

NODE NO.	REAL	IMAG.	MAG.
1	0.1400776E 01	-0.5437528E 00	0.1502610E 01
2	0.1317951E 01	-0.7555641E 00	0.1519168E 01
3	0.1270274E 01	-0.9699263E 00	0.1598234E 01
4	0.1426288E 01	-0.9693009E 00	0.1724483E 01
5	0.1651025E 01	-0.7896496E 00	0.1830144E 01
6	0.1726398E 01	-0.6227407E 00	0.1835280E 01
7	0.1938642E 01	-0.8654898E 00	0.2123064E 01
8	0.2013174E 01	-0.9156967E 00	0.2211644E 01
9	0.1924732E 01	-0.9932852E 00	0.2165919E 01
10	0.1443425E 01	-0.1179671E 01	0.1864161E 01
11	0.8872479E 00	-0.1221557E 01	0.1509771E 01
12	0.3849248E 00	-0.1067224E 01	0.1134519E 01
13	0.4556108E 00	-0.1108767E 01	0.1198726E 01
14	0.5439433E 00	-0.1161024E 01	0.1282127E 01
15	0.4434171E 00	-0.1095668E 01	0.1181993E 01
16	0.3681972E 00	-0.1049980E 01	0.1112666E 01
17	0.8534979E 00	-0.1194071E 01	0.1467741E 01
18	0.1331197E 01	-0.1064918E 01	0.1704739E 01
19	0.1582167E 01	-0.8777127E 00	0.1809318E 01
20	0.1510957E 01	-0.8338836E 00	0.1725790E 01

PTA = 0.1755839E 01
PTC = 0.1768724E 01

EXCITATIONS AND TRANSMISSION COEFFS.

0.0000E 00	0.0000E 00	0.5630E 00
0.0000E 00	0.4500E 02	0.4963E 00

TE GAIN AND FIELD PATTERNS

-0.9000E 02	0.9184E 00	0.9221E 00
-0.8800E 02	0.9186E 00	0.9222E 00
-0.8600E 02	0.9191E 00	0.9225E 00
-0.8400E 02	0.9200E 00	0.9229E 00
-0.8200E 02	0.9213E 00	0.9236E 00
-0.8000E 02	0.9229E 00	0.9244E 00
-0.7800E 02	0.9249E 00	0.9254E 00
-0.7600E 02	0.9272E 00	0.9265E 00
-0.7400E 02	0.9298E 00	0.9279E 00
-0.7200E 02	0.9328E 00	0.9293E 00
-0.7000E 02	0.9360E 00	0.9310E 00
-0.6800E 02	0.9396E 00	0.9327E 00
-0.6600E 02	0.9434E 00	0.9346E 00
-0.6400E 02	0.9475E 00	0.9367E 00
-0.6200E 02	0.9519E 00	0.9388E 00
-0.6000E 02	0.9565E 00	0.9411E 00
-0.5800E 02	0.9613E 00	0.9434E 00
-0.5600E 02	0.9662E 00	0.9458E 00
-0.5400E 02	0.9714E 00	0.9484E 00
-0.5200E 02	0.9766E 00	0.9509E 00
-0.5000E 02	0.9820E 00	0.9535E 00
-0.4800E 02	0.9875E 00	0.9562E 00
-0.4600E 02	0.9931E 00	0.9589E 00
-0.4400E 02	0.9987E 00	0.9616E 00
-0.4200E 02	0.1004E 01	0.9643E 00
-0.4000E 02	0.1010E 01	0.9670E 00
-0.3800E 02	0.1015E 01	0.9696E 00
-0.3600E 02	0.1021E 01	0.9722E 00
-0.3400E 02	0.1026E 01	0.9748E 00
-0.3200E 02	0.1032E 01	0.9773E 00
-0.3000E 02	0.1037E 01	0.9797E 00
-0.2800E 02	0.1042E 01	0.9821E 00
-0.2600E 02	0.1046E 01	0.9843E 00
-0.2400E 02	0.1051E 01	0.9864E 00
-0.2200E 02	0.1055E 01	0.9884E 00
-0.2000E 02	0.1059E 01	0.9903E 00
-0.1800E 02	0.1063E 01	0.9920E 00
-0.1600E 02	0.1066E 01	0.9936E 00
-0.1400E 02	0.1069E 01	0.9950E 00
-0.1200E 02	0.1072E 01	0.9962E 00
-0.1000E 02	0.1074E 01	0.9973E 00
-0.8000E 01	0.1076E 01	0.9982E 00
-0.6000E 01	0.1078E 01	0.9989E 00
-0.4000E 01	0.1079E 01	0.9995E 00
-0.2000E 01	0.1080E 01	0.9998E 00
0.0000E 00	0.1080E 01	0.1000E 01

0.2000E 01	0.1080E 01	0.1000E 01
0.4000E 01	0.1080E 01	0.9998E 00
0.6000E 01	0.1079E 01	0.9994E 00
0.8000E 01	0.1077E 01	0.9988E 00
0.1000E 02	0.1076E 01	0.9980E 00
0.1200E 02	0.1074E 01	0.9971E 00
0.1400E 02	0.1071E 01	0.9960E 00
0.1600E 02	0.1069E 01	0.9947E 00
0.1800E 02	0.1065E 01	0.9932E 00
0.2000E 02	0.1062E 01	0.9917E 00
0.2200E 02	0.1058E 01	0.9899E 00
0.2400E 02	0.1054E 01	0.9881E 00
0.2600E 02	0.1050E 01	0.9861E 00
0.2800E 02	0.1046E 01	0.9840E 00
0.3000E 02	0.1041E 01	0.9817E 00
0.3200E 02	0.1036E 01	0.9794E 00
0.3400E 02	0.1031E 01	0.9771E 00
0.3600E 02	0.1026E 01	0.9746E 00
0.3800E 02	0.1021E 01	0.9721E 00
0.4000E 02	0.1015E 01	0.9696E 00
0.4200E 02	0.1010E 01	0.9670E 00
0.4400E 02	0.1004E 01	0.9644E 00
0.4600E 02	0.9991E 00	0.9618E 00
0.4800E 02	0.9937E 00	0.9592E 00
0.5000E 02	0.9884E 00	0.9566E 00
0.5200E 02	0.9831E 00	0.9541E 00
0.5400E 02	0.9780E 00	0.9516E 00
0.5600E 02	0.9730E 00	0.9492E 00
0.5800E 02	0.9682E 00	0.9468E 00
0.6000E 02	0.9635E 00	0.9445E 00
0.6200E 02	0.9591E 00	0.9423E 00
0.6400E 02	0.9548E 00	0.9402E 00
0.6600E 02	0.9508E 00	0.9383E 00
0.6800E 02	0.9471E 00	0.9364E 00
0.7000E 02	0.9436E 00	0.9347E 00
0.7200E 02	0.9404E 00	0.9331E 00
0.7400E 02	0.9375E 00	0.9317E 00
0.7600E 02	0.9349E 00	0.9304E 00
0.7800E 02	0.9326E 00	0.9293E 00
0.8000E 02	0.9307E 00	0.9283E 00
0.8200E 02	0.9291E 00	0.9275E 00
0.8400E 02	0.9279E 00	0.9269E 00
0.8600E 02	0.9270E 00	0.9264E 00
0.8800E 02	0.9265E 00	0.9262E 00
0.9000E 02	0.9263E 00	0.9261E 00

EQUIVALENT CURRENTS FOR TM CASE

PLANE WAVE INCIDENCE. PHI = 0.000E 00

MAGNETIC CURRENTS...

NODE NO.	REAL	IMAG.	MAG.
2	-0.4254752E 00	-0.1147956E 01	0.1224267E 01
3	-0.5227808E 00	-0.1340040E 01	0.1438404E 01
4	-0.4851277E 00	-0.1245558E 01	0.1336699E 01
5	-0.3776911E 00	-0.1033994E 01	0.1100815E 01
13	-0.5729111E-01	-0.1250122E 00	0.1375147E 00
14	-0.7143658E-01	-0.1615084E 00	0.1766016E 00
15	-0.5633022E-01	-0.1232603E 00	0.1355219E 00

ELECTRIC CURRENTS TIMES ETA...

SEGMENT NO.	REAL	IMAG.	MAG.
1	0.9865695E 00	-0.3513402E 00	0.1047262E 01
2	0.1559619E 01	-0.5828362E 00	0.1664965E 01
3	0.1215407E 01	-0.4935932E 00	0.1311811E 01
4	0.1817409E 01	-0.6773867E 00	0.1939543E 01
5	0.9742750E 00	-0.3403842E 00	0.1032023E 01
6	-0.1604671E 01	0.5849431E 00	0.1707959E 01
7	-0.1626285E 00	0.5866562E-01	0.1728863E 00
8	-0.6836278E 00	0.2477846E 00	0.7271480E 00
9	-0.2209898E 01	0.8399488E 00	0.2364141E 01
10	-0.2017618E 01	0.7909234E 00	0.2167105E 01
11	-0.3933427E 00	0.1692539E 00	0.4282119E 00
12	-0.9100121E-01	0.1502394E-01	0.9223306E-01
13	-0.2705745E 00	0.7862997E-01	0.2817680E 00
14	-0.2669684E 00	0.7737666E-01	0.2779555E 00
15	-0.8381677E-01	0.1035898E-01	0.8445448E-01
16	-0.3690963E 00	0.1574180E 00	0.4012636E 00
17	-0.1987742E 01	0.7681251E 00	0.2130994E 01
18	-0.1142596E 01	0.4276093E 00	0.1219990E 01
19	-0.4231352E-01	0.7535454E-02	0.4297926E-01
20	-0.1795144E 01	0.6608344E 00	0.1912914E 01

PTA = 0.1068848E-01
PTC = 0.4894048E-02

EXCITATIONS AND TRANSMISSION COEFFS.

0.0000E 00	0.0000E 00	0.1558E-02
0.0000E 00	0.4500E 02	0.6828E-03

TM GAIN AND FIELD PATTERNS

-0.9000E 02	0.1860E-12	0.3019E-06
-0.8800E 02	0.2299E-02	0.3356E-01
-0.8600E 02	0.9187E-02	0.6710E-01
-0.8400E 02	0.2064E-01	0.1006E 00
-0.8200E 02	0.3661E-01	0.1339E 00
-0.8000E 02	0.5704E-01	0.1672E 00
-0.7800E 02	0.8186E-01	0.2003E 00
-0.7600E 02	0.1110E 00	0.2332E 00
-0.7400E 02	0.1442E 00	0.2659E 00
-0.7200E 02	0.1816E 00	0.2983E 00
-0.7000E 02	0.2228E 00	0.3304E 00
-0.6800E 02	0.2677E 00	0.3622E 00
-0.6600E 02	0.3162E 00	0.3937E 00
-0.6400E 02	0.3681E 00	0.4247E 00
-0.6200E 02	0.4231E 00	0.4553E 00
-0.6000E 02	0.4810E 00	0.4855E 00
-0.5800E 02	0.5416E 00	0.5152E 00
-0.5600E 02	0.6046E 00	0.5443E 00
-0.5400E 02	0.6697E 00	0.5728E 00
-0.5200E 02	0.7366E 00	0.6008E 00
-0.5000E 02	0.8050E 00	0.6281E 00
-0.4800E 02	0.8747E 00	0.6547E 00
-0.4600E 02	0.9452E 00	0.6806E 00
-0.4400E 02	0.1016E 01	0.7057E 00
-0.4200E 02	0.1088E 01	0.7300E 00
-0.4000E 02	0.1159E 01	0.7535E 00
-0.3800E 02	0.1229E 01	0.7762E 00
-0.3600E 02	0.1299E 01	0.7979E 00
-0.3400E 02	0.1368E 01	0.8187E 00
-0.3200E 02	0.1435E 01	0.8385E 00
-0.3000E 02	0.1500E 01	0.8572E 00
-0.2800E 02	0.1562E 01	0.8750E 00
-0.2600E 02	0.1622E 01	0.8917E 00
-0.2400E 02	0.1680E 01	0.9072E 00
-0.2200E 02	0.1733E 01	0.9217E 00
-0.2000E 02	0.1784E 01	0.9349E 00
-0.1800E 02	0.1830E 01	0.9470E 00
-0.1600E 02	0.1872E 01	0.9579E 00
-0.1400E 02	0.1910E 01	0.9675E 00
-0.1200E 02	0.1944E 01	0.9759E 00
-0.1000E 02	0.1972E 01	0.9831E 00
-0.8000E 01	0.1996E 01	0.9889E 00
-0.6000E 01	0.2014E 01	0.9935E 00
-0.4000E 01	0.2028E 01	0.9968E 00
-0.2000E 01	0.2036E 01	0.9988E 00
0.0000E 00	0.2041E 01	0.1000E 01

0.2000E 01	0.2036E 01	0.9988E 00
0.4000E 01	0.2028E 01	0.9968E 00
0.6000E 01	0.2014E 01	0.9935E 00
0.8000E 01	0.1996E 01	0.9890E 00
0.1000E 02	0.1972E 01	0.9831E 00
0.1200E 02	0.1944E 01	0.9760E 00
0.1400E 02	0.1911E 01	0.9676E 00
0.1600E 02	0.1873E 01	0.9579E 00
0.1800E 02	0.1830E 01	0.9471E 00
0.2000E 02	0.1784E 01	0.9350E 00
0.2200E 02	0.1734E 01	0.9217E 00
0.2400E 02	0.1680E 01	0.9073E 00
0.2600E 02	0.1623E 01	0.8917E 00
0.2800E 02	0.1563E 01	0.8751E 00
0.3000E 02	0.1500E 01	0.8573E 00
0.3200E 02	0.1435E 01	0.8385E 00
0.3400E 02	0.1368E 01	0.8187E 00
0.3600E 02	0.1299E 01	0.7980E 00
0.3800E 02	0.1230E 01	0.7763E 00
0.4000E 02	0.1159E 01	0.7536E 00
0.4200E 02	0.1088E 01	0.7301E 00
0.4400E 02	0.1017E 01	0.7058E 00
0.4600E 02	0.9455E 00	0.6807E 00
0.4800E 02	0.8749E 00	0.6548E 00
0.5000E 02	0.8053E 00	0.6282E 00
0.5200E 02	0.7368E 00	0.6009E 00
0.5400E 02	0.6699E 00	0.5729E 00
0.5600E 02	0.6047E 00	0.5444E 00
0.5800E 02	0.5418E 00	0.5152E 00
0.6000E 02	0.4812E 00	0.4856E 00
0.6200E 02	0.4232E 00	0.4554E 00
0.6400E 02	0.3682E 00	0.4248E 00
0.6600E 02	0.3163E 00	0.3937E 00
0.6800E 02	0.2678E 00	0.3623E 00
0.7000E 02	0.2228E 00	0.3305E 00
0.7200E 02	0.1816E 00	0.2983E 00
0.7400E 02	0.1443E 00	0.2659E 00
0.7600E 02	0.1110E 00	0.2332E 00
0.7800E 02	0.8188E-01	0.2003E 00
0.8000E 02	0.5706E-01	0.1672E 00
0.8200E 02	0.3662E-01	0.1340E 00
0.8400E 02	0.2064E-01	0.1006E 00
0.8600E 02	0.9188E-02	0.6710E-01
0.8800E 02	0.2299E-02	0.3356E-01
0.9000E 02	0.1606E-09	0.8871E-05

2.1.3 Minimum Storage Requirements for Arrays

The minimum storage required for each array used in Program I is given here. Arrays which are execution-time dimensioned are not mentioned (i.e., array UL of subroutine DECOMP). Integers which are used to specify the array sizes are defined by:

ND(i) = Node number N_i where i=1,2,3,4,5.

NGQ = Order of Gaussian quadrature formula used for numerical integrations.

NU(1) = Number of elements in $\vec{V}^1$.

NU(2) = Number of elements in $\vec{V}^3$.

NU(3) = Number of elements in $\eta_o \vec{I}$.

NTU = NU(1) + NU(2) + NU(3) = total number of unknowns in system of equations (2-17).

NEX = Number of excitations.

The allocations required for the arrays in common blocks, which appear in several program segments, are given by:

```
     CUV:  DIMENSION ULX(ND(5)), ULY(ND(5)), ND(5)
       C:  DIMENSION RCX(ND(5)), RCY(ND(5)), DC(ND(5))
      CK:  COMPLEX RXK(ND(5)), RYK(ND(5)), DCK(ND(5))
     GQI:  DIMENSION A(NGQ), T(NGQ)
```

The allocations required for arrays used in subroutines are:

```
MAIN PGM:  DIMENSION PHI(2*NEX), IJOB(6)
    SOLN:  COMPLEX Y(NTU*NTU), IX(NU(1)*NEX),
           ANS(NTU), X(NTU), YHS(NU(1)),
           VM(NTU*NEX)
```

```
               DIMENSION IJOB(6), IPS(NTU), NU(3),
               ND(5), PHI(2*NEX), PIN(NEX), PT(NEX)
      PCK:     COMPLEX X(NTU)
TEMAT and
    TMMAT:     COMPLEX YD1((NU(1) + NU(2))*NU(3)),
               YHS(NU(1)), IHAT(NEX*NU(1))
               DIMENSION PHI(2*NEX), PIN(NEX),
               NU(3)
    GAIN1:     COMPLEX FT((ND(4)-ND(3)+1)*NI),
               XI(NTU*NEX), DCK(ND(4)-ND(3)+1),
               RYK(ND(4)-ND(3)+1),
               DIMENSION PHI(2*NEX), FP(NI),
               GA(NI), PT(NEX), NU(3) [Note: NI is defined on p. 7]
   TRANS1:     COMPLEX YHS(NU(1)), YAUX(NU(1)),
               VC(NU(1)), VM(NTU*NEX)
               DIMENSION PHI(2*NEX), PIN(NEX),
               PT(NEX), NU(3)
TMEXC and
    TEEXC:     COMPLEX IX(NU(1)*NEX)
               DIMENSION PHI(2*NEX), PIN(NEX), ND(5)
     P2Q2:     COMPLEX Y((NU(1)+NU(2))*NU(3))
               DIMENSION NU(3), ND(5)
   DECOMP:     DIMENSION SCL(NTU), IPS(NTU)
    SOLVE:     DIMENSION IPS(NTU)
```

2.2 Subroutine CDATA

The necessary argument parameter here is WAVNO, the wavenumber of free space. This subroutine reads the second sequence of data cards described in Section 2.1.1. The first card contains the node numbers N_i which specify the beginning and end points of lines Γ_i in terms of the number of straight line segments which comprise them. These numbers are stored in array ND and are arranged such that $N_1 = 1$ denotes the origin (where Γ_1 and Γ_4 meet). This point is also represented by N_5 because C is a closed contour. Each line Γ_i is broken up into $N_{i+1} - N_i$ subsections. The next data card read in contains the y coordinate of node N_1 which is zero. Since lines Γ_1 and Γ_3 are parallel to the y axis, only one coordinate is used to specify points along them.

Quantities computed and stored in common blocks are the tangential unit vector for each line segment (stored in ULX and ULY), the length of each line segment (stored in DC), and a vector from the origin to the midpoint of each line segment (stored in RCX and RCY). These last three are all multiplied by a factor of k_o in DO loop 6.

It is helpful, when using the programs, to draw the slit cross section labelling the coordinates of the endpoints of each straight line segment as shown in Fig. 3. Then the numbers which go on the required data cards can be written down directly.

```
      SUBROUTINE CDATA(WAVNO)
      COMMON /CUV/ULX(60),ULY(60),ND(5)/C/RCX(60),RCY(60),DC(60)
100   FORMAT(6I5)
102   FORMAT(6E11.4)
      READ(1,100)(ND(I),I=1,5)
      WRITE(3,108) ND
      MF=1
      D=0.
      READ(1,102) Y1
```

```
      DO 5  J=1,3,2
      N1=ND(J)+1
      N2=ND(J+1)
      S=0.
      DO 1  I=N1,N2
      READ(1,102) Y2
      RCX(I-1)=D
      RCY(I-1)=(Y1+Y2)/2.
      DC(I-1)=ABS(Y2-Y1)
      ULY(I-1)=MF
      ULX(I-1)=0.
      Y1=Y2
      S=S+DC(I-1)
1     CONTINUE
      IF(J.EQ.1) WRITE(3,101) S
      IF(J.EQ.3) WRITE(3,104) S
      X1=D
      N1=ND(J+1)+1
      N2=ND(J+2)
      DO 2  I=N1,N2
      READ(1,102) X2,Y2
      RCX(I-1)=(X1+X2)/2.
      RCY(I-1)=(Y1+Y2)/2.
      AX=X2-X1
      AY=Y2-Y1
      DS=SQRT(AX*AX+AY*AY)
      ULX(I-1)=AX/DS
      ULY(I-1)=AY/DS
      DC(I-1)=DS
      X1=X2
      Y1=Y2
2     CONTINUE
      D=X2
      MF=-MF
      IF(J.EQ.1) WRITE(3,103) D
5     CONTINUE
101   FORMAT('1','WIDTH OF SLIT IN REGION A=',E11.4)
103   FORMAT('-','THICKNESS OF GROUND PLANE =',E11.4)
104   FORMAT('-','WIDTH OF SLIT IN REGION C =',E11.4)
105   FORMAT('-','INPUT DATA DEFINING CONTOUR...')
106   FORMAT('-','NODE',5X,'ULX',8X,'ULY',8X,'RCX',8X,'RCY',
     18X,'DC')
107   FORMAT(' ',I5,5E11.4)
108   FORMAT('-','ARRAY ND =',5I5)
      N1=ND(5)-1
      WRITE(3,105)
      WRITE(3,106)
      DO 6  I=1,N1
      WRITE(3,107) I,ULX(I),ULY(I),RCX(I),RCY(I),DC(I)
      RCX(I)=RCX(I)*WAVNO
      RCY(I)=RCY(I)*WAVNO
      DC(I)=DC(I)*WAVNO
6     CONTINUE
      RETURN
      END
```

2.3 Function Subprograms CHO2 and CH12

These function subprograms return the Hankel functions of the second kind, orders zero and one, for a complex argument as

$$CHO2(z) = H_0^{(2)}(z)$$

$$CH12(z) = H_1^{(2)}(z)$$

where z is a complex number. The polynomial approximations of Abramowitz and Stegun [2, 9.4.1-9.4.6] for real z are used. It is assumed that these polynomial representations may be analytically continued into the complex plane by replacing real z by complex z [3]. As $|z|$ becomes large, computations using these polynomials may become unreliable as pointed out in [3]. Thus the polynomial approximation is used when $|z| \leq 12$ and the asymptotic expansion for the Hankel function [2, 9.2.8-9.2.10] is used when $|z| > 12$. Enough terms in the asymptotic series are taken until the magnitude of a term is less than some positive constant EPS.

```
      COMPLEX FUNCTION CHO2(Z)
      COMPLEX U/(0.,1.)/,Z,BSJO,BSYO,Y,W,PO,FO,F,CEXP
      COMPLEX CCOS,CSIN,CSQRT,CLOG,SPO,SQO,QO
      PI=3.141593
      EPS=.1E-07
      A=CABS(Z)
      IF(A.GE.12.) GO TO 5
      BSJO=(1.,0.)
      IF(A.EQ.0.) GO TO 2
      IF(A.GT.3.) GO TO 1
      Y=Z*Z/9.
      BSJO=1.+Y*(-2.2499997+Y*(1.2656208+Y*(-.3163866
     1+Y*(.0444479+Y*(-.0039444+Y*.00021)))))
      GO TO 2
1     W=3./Z
      FO=.79788456+W*(-.00000077+W*(-.0055274+W*(-.00009512
     1+W*(.00137237+W*(-.00072805+W*.00014476)))))
```

```
      P0=.78539816+W*(.04166397+W*(.00003954+W*(-.00262573
     1+W*(.00054125+W*(.00029333-W*.00013558)))))
      BSJ0=F0*CCOS(Z-P0)/CSQRT(Z)
    2 CONTINUE
      BSY0=(-1.E75,0.)
      IF(A.EQ.0.) GO TO 3
      IF(A.GT.3.) GO TO 4
      BSY0=.63661977*CLOG(.5*Z)*BSJ0+.36746691+Y*(.60559366
     1+Y*(-.74350384+Y*(.25300117+Y*(-.04261214+Y*(.00427916
     2-Y*.00024846)))))
      GO TO 3
    4 CONTINUE
      BSY0=F0*CSIN(Z-P0)/CSQRT(Z)
    3 CONTINUE
      CH02=BSJ0-U*BSY0
      RETURN
    5 W=8.*Z
      Y=W*W
      SP0=-9./(2.*Y)
      P0=1.+SP0
      DO 6 I=1,10
      K1=4*I+1
      K2=K1+2
      K3=2*I+1
      SP0=-SP0*K1*K1*K2*K2/(K3*(K3+1)*Y)
      P0=P0+SP0
      IF(CABS(SP0).LE.EPS) GO TO 7
    6 CONTINUE
    7 SQ0=225./(6.*Y*W)
      Q0=-1./W+SQ0
      DO 8 I=1,10
      K1=4*I+3
      K2=K1+2
      K3=2*I+2
      SQ0=-SQ0*K1*K1*K2*K2/(K3*(K3+1)*Y)
      Q0=Q0+SQ0
      IF(CABS(SQ0).LE.EPS) GO TO 9
    8 CONTINUE
    9 F=CSQRT(2.*U/PI/Z)*CEXP(-U*Z)
      CH02=F*(P0-U*Q0)
      RETURN
      END


      COMPLEX FUNCTION CH12(Z)
      COMPLEX U/(0.,1.)/,Z,BSJ1,BSY1,Y,W,P1,F1,CEXP
      COMPLEX CCOS,CSQRT,CLOG,CSIN,SP1,SQ1,Q1,F
      PI=3.141593
      EPS=.1E-07
      A=CABS(Z)
      IF(A.GE.12.) GO TO 5
```

```
      BSJ1=0.
      IF(A.EQ.0.) GO TO 2
      IF(A.GT.3.) GO TO 1
      Y=Z*Z/9.
      BSJ1=Z*(.5+Y*(-.56249985+Y*(.21093573+Y*(-.03954289
     1+Y*(.00443319+Y*(-.00031761+Y*.00001109))))))
      GO TO 2
 1    W=3./Z
      F1=.79788456+W*(.00000156+W*(.01659667+W*(.00017105
     1+W*(-.00249511+W*(.00113653-W*.00020033)))))
      P1=2.35619449+W*(-.12499612+W*(-.00005650+W*(.00637879
     1+W*(-.00074348+W*(-.00079824+W*.00029166)))))
      BSJ1=F1*CCOS(Z-P1)/CSQRT(Z)
 2    CONTINUE
      BSY1=(-1.E75,0.)
      IF(A.EQ.0.) GO TO 3
      IF(A.GT.3.) GO TO 4
      BSY1=(-.63661977+Y*(.2212091+Y*(2.1682709+Y*(-1.3164827
     1+Y*(.3123951+Y*(-.0400976+Y*.0027873))))))/Z
     2+.63661977*CLOG(.5*Z)*BSJ1
      GO TO 3
 4    BSY1=F1*CSIN(Z-P1)/CSQRT(Z)
 3    CONTINUE
      CH12=BSJ1-U*BSY1
      RETURN
 5    W=8.*Z
      Y=W*W
      SP1=15./(2.*Y)
      P1=1.+SP1
      DO 6 I=1,10
      K1=4-(4*I+1)**2
      K2=4-(4*I+3)**2
      K3=2*I+1
      SP1=-SP1*K1*K2/(K3*(K3+1)*Y)
      P1=P1+SP1
      IF(CABS(SP1).LE.EPS) GO TO 7
 6    CONTINUE
 7    SQ1=-105./(2.*Y*W)
      Q1=3./W+SQ1
      DO 8 I=1,10
      K1=4-(4*I+3)**2
      K2=4-(4*I+5)**2
      K3=2*I+2
      SQ1=-SQ1*K1*K2/(K3*(K3+1)*Y)
      Q1=Q1+SQ1
      IF(CABS(SQ1).LE.EPS) GO TO 9
 8    CONTINUE
 9    F=U*CSQRT(2.*U/PI/Z)*CEXP(-U*Z)
      CH12=F*(P1-U*Q1)
      RETURN
      END
```

2.4 Subroutines TMEXC and TEEXC

The subroutines compute the excitation vector $\vec{I}^i$, given by Eq. (2-28), for the TM case (TMEXC) or the TE case (TEEXC). The argument parameters are defined by:

Input: NU1 = number of elements in $\vec{I}^i$.

NEX = number of excitations.

ETA = η_a/η_o

KA = k_a/k_o

WAVNO = k_o

PHI = excitation array defined in Section 2.1.1.

ND = node array defined in section 2.1.1.

Output: IX = array containing vector $\vec{I}^i$ for all excitations.

PIN = array containing the time average power times $k_o\eta_o$ intercepted by the aperture Γ_1 when the source is normally incident for each excitation.

The possible excitations here consist of a plane wave incident at an angle ϕ^i measured with respect to the negative x axis and a line source placed at coordinates (x_s, y_s) in region a. The coordinates of Fig. 1 are used where $-\infty < y_s < \infty$, $x_s < 0$, and $-90° \leq \phi^i \leq 90°$. The strength of each excitation is adjusted so that the electric field is unity and has zero phase at the center of the illuminated aperture face Γ_1.

DO loops 1 and 5 compute the elements of $\vec{I}^i$ for plane wave incidence and use Eq. (3-40) for the TE case and (4-33) for the TM case. DO loop 6 computes the elements of $\vec{I}^i$ for line source incidence and uses

Eq. (3-43) for the TE case and (4-36) for the TM case. Here a Gaussian quadrature formula is used for the integrations. The time average power intercepted by Γ_1 when the source is normally incident, denoted by P_{iN} in [1] and given by Eqs. (5-14) and (5-15) for the TE case and (5-18) and (5-19) for the TM case, is stored in array PIN as

$$PIN(j) = k_o \eta_o P_{iN}$$

where P_{iN} is computed for the jth excitation and j = 1,2,..., NEX. This computation is used later in subroutine TRANS1 for normalizing the transmission coefficient.

```
      SUBROUTINE TMEXC(IX,NU1,NEX,ETA,KA,WAVNO,PHI,PIN,ND)
      COMPLEX CEXP,U,USPHI,RXK,RYK,DCK,WA2,CH02,CH12
      COMPLEX IX(500),ETA,KA,ARG1,FE,RS,WA
      COMPLEX ARG2,SUM1,SUM2,C1,C2,CSQRT ,XS,YS
      COMMON /CK/RXK(60),RYK(60),DCK(60)/GQI/A(10),T(10),NGQ
      DIMENSION PHI(100),PIN(100),ND(5)
      DATA PI/3.141593/,U/(0.,1.)/
      WA2=(RYK(ND(2)-1)+DCK(ND(2)-1)/2.
     1+RYK(ND(1))-DCK(ND(1))/2.)/2.
      WA=RYK(ND(2)-1)+DCK(ND(2)-1)/2.-RYK(ND(1))+DCK(ND(1))/2.
      K=1
      KC=1
      DO 1 J=1,NEX
      IF(PHI(K).NE.0.) GO TO 10
      IF(PHI(K+1).EQ.0.) GO TO 3
      CPHI=COS(PHI(K+1)*PI/180.)
      USPHI=U*SIN(PHI(K+1)*PI/180.)
      FE=CPHI/(ETA*KA*USPHI)
      DO 2 I=1,NU1
      C1=DCK(I)*USPHI
      C2=DCK(I+1)*USPHI
      ARG1=(RYK(I)+DCK(I)/2.-WA2)*USPHI
      IX(KC)=-FE*CEXP(ARG1)*((CEXP(-C1)-1.)/C1+(CEXP(C2)-1.)/C2)
      KC=KC+1
2     CONTINUE
      GO TO 4
3     DO 5 I=1,NU1
      IX(KC)=-(DCK(I)+DCK(I+1))/(2.*KA*ETA)
      KC=KC+1
5     CONTINUE
4     PIN(J)=REAL(WA/(ETA*KA))
```

```
      GO TO 11
10    CONTINUE
      XS=WAVNO*KA*PHI(K)
      YS=WAVNO*KA*PHI(K+1)
      RS=CSQRT(XS*XS+(WA2-YS)**2)
      FE=CH02(RS)
      DO 6 I=1,NU1
      SUM1=0.
      SUM2=0.
      DO 7 L=1,NGQ
      ARG1=CSQRT(XS*XS+(DCK(I)*T(L)+RYK(I)-YS)**2)
      ARG2=CSQRT(XS*XS+(DCK(I+1)*T(L)+RYK(I+1)-YS)**2)
      SUM1=SUM1+A(L)*(T(L)+.5)*CH12(ARG1)/ARG1
      SUM2=SUM2+A(L)*(.5-T(L))*CH12(ARG2)/ARG2
7     CONTINUE
      IX(KC)=XS*(DCK(I)*SUM1+DCK(I+1)*SUM2)/(ETA*KA*U*FE)
      KC=KC+1
6     CONTINUE
      THET=2.*ATAN(REAL(WA/(2.*RS)))
      PIN(J)=2.*THET/REAL(PI*ETA*KA*CABS(FE)**2)
11    CONTINUE
      K=K+2
1     CONTINUE
      RETURN
      END


      SUBROUTINE TEEXC(IX,NU1,NEX,ETA,KA,WAVNO,PHI,PIN,ND)
      COMPLEX IX(500),ETA,KA,FE,ARG,SUM,RS,XS,YS,U,WA
      COMPLEX CH02,CH12,CSIN,RXK,RYK,DCK,WA2,CSQRT,CEXP
      COMMON /CK/RXK(60),RYK(60),DCK(60)/GQI/A(10),T(10),NGQ
      DIMENSION PHI(100),PIN(100),ND(5)
      DATA PI/3.141593/,U/(0.,1.)/
      WA2=(RYK(ND(2)-1)+DCK(ND(2)-1)/2.
     1+RYK(ND(1))-DCK(ND(1))/2.)/2.
      WA=RYK(ND(2)-1)+DCK(ND(2)-1)/2.-RYK(ND(1))+DCK(ND(1))/2.
      K=1
      KC=1
      DO 1 J=1,NEX
      IF(PHI(K).NE.0.) GO TO 10
      IF(PHI(K+1).EQ.0.) GO TO 3
      SPHI=SIN(PHI(K+1)*PI/180.)
      DO 2 I=1,NU1
      ARG=DCK(I)*SPHI/2.
      FE=DCK(I)*CEXP(U*SPHI*(RYK(I)-WA2))/(ETA*KA)
      IX(KC)=FE*CSIN(ARG)/ARG
      KC=KC+1
2     CONTINUE
```

```
      GO TO 4
3     DO 5 I=1,NU1
      IX(KC)=DCK(I)/(KA*ETA)
      KC=KC+1
5     CONTINUE
4     PIN(J)=REAL(WA/(ETA*KA))
      GO TO 11
10    CONTINUE
      XS=WAVNO*KA*PHI(K)
      YS=WAVNO*KA*PHI(K+1)
      RS=CSQRT(XS*XS+(WA2-YS)**2)
      FE=CH12(RS)
      DO 6 I=1,NU1
      SUM=0.
      DO 7 L=1,NGQ
      ARG=CSQRT(XS*XS+(DCK(I)*T(L)+RYK(I)-YS)**2)
      SUM=SUM+A(L)*CH02(ARG)
7     CONTINUE
      IX(KC)=U*DCK(I)*SUM/(ETA*KA*FE)
      KC=KC+1
6     CONTINUE
      THET=2.*ATAN(REAL(WA/(2.*RS)))
      PIN(J)=2.*THET/REAL(PI*ETA*KA*CABS(FE)**2)
11    CONTINUE
      K=K+2
1     CONTINUE
      RETURN
      END
```

2.5 Subroutine P2Q2

This subroutine computes the inner product P_2 of Eq. (3-18) for the TE case or inner product Q_2 of Eq. (4-16) for the TM case. The argument parameters are defined by

Input: NU = array containing numbers of unknowns M^1, M^3, and J^b.

ND = node array defined in 2.1.1.

ID = integer option variable, ID = 1 for TE case and ID = 2 for TM case.

Output: Y = array containing the submatrix

$$\begin{bmatrix} [U^1] \\ [U^3] \end{bmatrix}$$

of Eq. (2-17).

The array Y is initialized to zero and the computation is quite straightforward. The elements are stored columnwise.

```
      SUBROUTINE P2Q2(Y,NU,ND,ID)
      COMPLEX Y(1000)
      INTEGER NU(3),ND(5)
      COMMON /C/RCX(60),RCY(60),DC(60)
      NU1=NU(1)
      NU2=NU(2)
      NU3=NU(3)
      NR=NU1+NU2
      K=1
      DO 1 IC=1,NU3
      DO 1 IR=1,NR
      Y(K)=0.
      K=K+1
1     CONTINUE
C--------ID=1...TE U MATRIX TO BE COMPUTED
C--------ID=2...TM U MATRIX TO BE COMPUTED
      MF=3-2*ID
```

```
      K=1
      DO 2 I=1,NU1
      Y(K)=MF*DC(I)/4.
      Y(K+NR)=MF*DC(I+ID-1)/4.
      K=K+1+NR
2     CONTINUE
      ND3=ND(3)-1
      K=NR*ND3+NU1+1
      DO 3 I=1,NU2
      Y(K)=MF*DC(I+ND3)/4.
      Y(K+NR)=MF*DC(I+ND3+ID-1)/4.
      K=K+1+NR
3     CONTINUE
      RETURN
      END
```

2.6 Function Subprograms P1, P6S, Q5, P5, and ALPHA

These function subprograms are quite straightforward. P1, P6S, Q5, and P5 all return the result of a symmetric product, where the integral has been approximated by a Gaussian quadrature rule. Their use is succinctly described by:

$$P1(ICM,ICN) = \frac{k_a \eta_a}{k_o \eta_o} P_1 \quad \text{and} \quad P_1 \text{ is given by Eq. (3-16)}$$

where

ICM = m

ICN = n

$$P6S(LCM,LCN,ISM,ISN) = S_6(m,n,p,q) \text{ of Eq. (3-37)}$$

where

LCM = m

LCN = n

ISM = p

ISN = q

Q5(ICM,ICN) = $S_1 + S_2$ which are given by Eqs. (4-29) and (4-30) where

ICM = m

ICN = n

P5(ICM, ICN) = $S_1 + S_2$ which are given by Eqs. (3-31) and (3-32) where

ICM = m

ICN = n

ALPHA(z) = $\alpha(z)$ of Eq. (3-17) where z is a complex number. This integral is computed by Struve functions [2, 11.1.7], the result being given by

$$\alpha(z) = z\, H_0^{(2)}(z) + z[H_0(z)\, H_1^{(2)}(z) - H_1(z)\, H_0^{(2)}(z)]$$

where

$$H_0(z) \approx z - \frac{z^3}{1^2.3^2} + \frac{z^5}{1^2.3^2.5^2} - \frac{z^7}{1^2.3^2.5^2.7^2}$$

$$H_1(z) \approx \frac{z^2}{1^2.3} - \frac{z^4}{1^2.3^2.5} + \frac{z^6}{1^2.3^2.5^2.7} - \frac{z^8}{1^2.3^2.5^2.7^2.9}$$

```
      COMPLEX FUNCTION P1(ICM,ICN)
      COMMON /CUV/ULX(60),ULY(60),ND(5)/CK/RXK(60),RYK(60),DCK(60)
      COMMON /GQI/A(10),T(10),NGQ
      COMPLEX RXK,RYK,DCK,DCM,DCN,RMNX,RMNY
      COMPLEX CH02,SUM,AX,AY,CSQRT,ARG,ALPHA
      SUM=0.
      DCM=DCK(ICM)
      DCN=DCK(ICN)
      IF(ICM.EQ.ICN) GO TO 2
      RMNX=RXK(ICM)-RXK(ICN)
      RMNY=RYK(ICM)-RYK(ICN)
      DO 1 I=1,NGQ
      DO 1 J=1,NGQ
```

```
      AX=DCM*T(I)*ULX(ICM)-DCN*T(J)*ULX(ICN)+RMNX
      AY=DCM*T(I)*ULY(ICM)-DCN*T(J)*ULY(ICN)+RMNY
      ARG=CSQRT(AX*AX+AY*AY)
      SUM=SUM+A(I)*A(J)*CH02(ARG)
1     CONTINUE
      P1=SUM*DCM*DCN/4.
      RETURN
2     DO 3 I=1,NGQ
      F1=T(I)+.5
      F2=.5-T(I)
      SUM=SUM+A(I)*(ALPHA(DCM*F1)+ALPHA(DCM*F2))
3     CONTINUE
      P1=DCM*SUM/4.
      RETURN
      END


      COMPLEX FUNCTION P6S(LCM,LCN,ISM,ISN)
      COMMON /CUV/ULX(60),ULY(60),ND(5)/CK/RXK(60),RYK(60),DCK(60)
      COMMON /GGI/A(10),T(10),NGQ
      COMPLEX RXK,RYK,DCK,DCM,DCN,ARG,ALPHA,RMNX,RMNY
      COMPLEX CH02,CH12,SUM,AX,AY,CSQRT,DD,F
      ICM=LCM
      ICN=LCN
      IF(LCM.EQ.0) ICM=ND(5)-1
      IF(LCN.EQ.0) ICN=ND(5)-1
      SUM=0.
      DCM=DCK(ICM)
      DCN=DCK(ICN)
      DD=ISM*ISN/DCM/DCN
      IF(ICM.EQ.ICN) GO TO 2
      RMNX=RXK(ICM)-RXK(ICN)
      RMNY=RYK(ICM)-RYK(ICN)
      DOT=ULX(ICM)*ULX(ICN)+ULY(ICM)*ULY(ICN)
      DO 1 I=1,NGQ
      DO 1 J=1,NGQ
      AX=DCM*T(I)*ULX(ICM)-DCN*T(J)*ULX(ICN)+RMNX
      AY=DCM*T(I)*ULY(ICM)-DCN*T(J)*ULY(ICN)+RMNY
      ARG=CSQRT(AX*AX+AY*AY)
      F=DOT*(ISM*T(I)+.5)*(ISN*T(J)+.5)-DD
      SUM=SUM+A(I)*A(J)*F*CH02(ARG)
1     CONTINUE
      P6S=SUM*DCM*DCN/4.
      RETURN
2     DO 3 I=1,NGQ
      F1=T(I)+.5
      F2=.5-T(I)
      AX=((ISM*T(I)+.5)*(ISN*T(I)+.5)-DD)*
     1(ALPHA(F1*DCN)+ALPHA(F2*DCN))
      AY=ISN*(ISM*T(I)+.5)*(F2*CH12(F2*DCN)-F1*CH12(F1*DCN))
      SUM=SUM+A(I)*(AX+AY)
3     CONTINUE
      P6S=DCM*SUM/4.
      RETURN
      END
```

```
      COMPLEX FUNCTION Q5(ICM,ICN)
      COMMON /CUV/ULX(60),ULY(60),ND(5)/CK/RXK(60),RYK(60),DCK(60)
      COMMON /GQI/A(10),T(10),NGQ
      COMPLEX RXK,RYK,DCK,DCM,DCN,DCN1,ARG,RMNX
      COMPLEX RMNX1,RMNY,RMNY1
      COMPLEX CH12,SUM,S1,S2,CSQRT,AX,AY,DOT,U
      DATA U/(0.,1.)/
      N1=ICN-1
      DCM=DCK(ICM)
      IF(ICN.EQ.1) N1=ND(5)-1
      DCN1=DCK(N1)
      DCN=DCK(ICN)
      RMNX=RXK(ICM)-RXK(ICN)
      RMNX1=RXK(ICM)-RXK(N1)
      RMNY=RYK(ICM)-RYK(ICN)
      RMNY1=RYK(ICM)-RYK(N1)
      IF(ICM.NE.N1) GO TO 3
      S1=.25
      GO TO 5
3     S1=0.
      DO 4 I=1,NGQ
      DO 4 J=1,NGQ
      AX=RMNX1+T(I)*DCM*ULX(ICM)-T(J)*DCN1*ULX(N1)
      AY=RMNY1+T(I)*DCM*ULY(ICM)-T(J)*DCN1*ULY(N1)
      DOT=-ULY(N1)*AX+ULX(N1)*AY
      ARG=CSQRT(AX*AX+AY*AY)
      S1=S1+A(I)*A(J)*(T(J)+.5)*DOT*CH12(ARG)/ARG
4     CONTINUE
      S1= DCN1*S1/4./U
      IF(ICM.NE.ICN) GO TO 5
      S2=.25
      GO TO 6
5     S2=0.
      DO 7 I=1,NGQ
      DO 7 J=1,NGQ
      AX=RMNX+T(I)*DCM*ULX(ICM)-T(J)*DCN*ULX(ICN)
      AY=RMNY+T(I)*DCM*ULY(ICM)-T(J)*DCN*ULY(ICN)
      DOT=-ULY(ICN)*AX+ULX(ICN)*AY
      ARG=CSQRT(AX*AX+AY*AY)
      S2=S2+A(I)*A(J)*(-T(J)+.5)*DOT*CH12(ARG)/ARG
7     CONTINUE
      S2= DCN*S2/4./U
6     Q5=DCM*(S1+S2)
      RETURN
      END


      COMPLEX FUNCTION P5(ICM,ICN)
      COMMON /CUV/ULX(60),ULY(60),ND(5)/CK/RXK(60),RYK(60),DCK(60)
      COMMON /GQI/A(10),T(10),NGQ
      COMPLEX RXK,RYK,DCK,DCM,DCN,DCM1,ARG,RMNX
      COMPLEX RMNX1,RMNY,RMNY1
      COMPLEX CH12,SUM,S1,S2,CSQRT,AX,AY,DOT,U
      DATA U/(0.,1.)/
```

```
      M1=ICM-1
      DCN=DCK(ICN)
      IF(ICM.EQ.1) M1=ND(5)-1
      DCM1=DCK(M1)
      DCM=DCK(ICM)
      RMNX=RXK(ICM)-RXK(ICN)
      RMNX1=RXK(M1)-RXK(ICN)
      RMNY=RYK(ICM)-RYK(ICN)
      RMNY1=RYK(M1)-RYK(ICN)
      IF(M1.NE.ICN) GO TO 3
      S1=-.25
      GO TO 5
3     S1=0.
      DO 4 I=1,NGQ
      DO 4 J=1,NGQ
      AX=RMNX1+T(I)*DCM1*ULX(M1)-T(J)*DCN*ULX(ICN)
      AY=RMNY1+T(I)*DCM1*ULY(M1)-T(J)*DCN*ULY(ICN)
      DOT=-ULY(M1)*AX+ULX(M1)*AY
      ARG=CSQRT(AX*AX+AY*AY)
      S1=S1+A(I)*A(J)*(T(I)+.5)*DOT*CH12(ARG)/ARG
4     CONTINUE
      S1=-DCM1*S1/4./U
      IF(ICM.NE.ICN) GO TO 5
      S2=-.25
      GO TO 6
5     S2=0.
      DO 7 I=1,NGQ
      DO 7 J=1,NGQ
      AX=RMNX+T(I)*DCM*ULX(ICM)-T(J)*DCN*ULX(ICN)
      AY=RMNY+T(I)*DCM*ULY(ICM)-T(J)*DCN*ULY(ICN)
      DOT=-ULY(ICM)*AX+ULX(ICM)*AY
      ARG=CSQRT(AX*AX+AY*AY)
      S2=S2+A(I)*A(J)*(-T(I)+.5)*DOT*CH12(ARG)/ARG
7     CONTINUE
      S2=-DCM*S2/4./U
6     P5=DCN*(S1+S2)
      RETURN
      END
```

```
      COMPLEX FUNCTION ALPHA(Z)
      COMPLEX CH02,CH12,Z,H0,H1,Z2
      Z2=Z*Z
      H0=Z*(1.-Z2/9.*(1.-Z2/25.*(1.-Z2/49.)))
      H1=(Z2/3.)*(1.-Z2/15.*(1.-Z2/35.*(1.-Z2/63.)))
      ALPHA=Z*CH02(Z)*(1.-H1)+Z*CH12(Z)*H0
      RETURN
      END
```

2.7 Subroutines TEMAT and TMMAT

These subroutines form the coefficient matrix and excitation vectors of Eq. (2-17) for the TE case (TEMAT) or the TM case (TMMAT). The argument parameters are defined by:

Input: NU = array containing number of elements in vectors $\vec{V}^1$, $\vec{V}^3$, and $\eta_o\vec{I}$.

NN = number of elements in coefficient matrix of (2-17).

NEX = number of excitation sources to be considered.

PHI = excitation array defined in 2.1.1.

Output: Y = array containing elements of coefficient matrix of (2-17) stored columnwise.

IHAT = excitation matrix, each column of which contains an excitation vector of (2-17).

PIN = array containing normal incident power for each excitation.

YHS = array containing elements of Y^c stored columnwise.

The array Y is initialized to zero. The elements of $\vec{I}^i$ for each excitation are found by calling TEEXC for the TE case or TMEXC for the TM case. The various submatrices of Eq. (2-17) are created and stored in their appropriate places in Y by the following sequence:

Y^a	DO loop 3 and subroutine SYM
Y^c	DO loop 5 and subroutine SYM
U^1, U^3	Subroutine P2Q2, DO loop 6.
Y^{e1}	DO loop 8
Y^{e3}	DO loop 9
T^e	DO loops 10 and 11 and subroutine SYM

The matrix Y^c is stored in the dummy array YHS for later use in computing the transmitted power in subroutine TRANS1. Here the matrices Y^a and Y^c are divided into subsections of equal length. If it is desired to use subsections of unequal length on these aperture faces, DO loops 3 and 5, as well as the calls to SYM, must be changed slightly.

```
      SUBROUTINE TEMAT(Y,NU,NN,IHAT,NEX,PHI,PIN,YHS)
      COMPLEX Y(NN),YD1(1000),P1,P5,P6S,ETK,YHS(30)
      COMMON /CUV/ULX(60),ULY(60),ND(5)/C/RCX(60),RCY(60),DC(60)
      COMMON /CK/RXK(60),RYK(60),DCK(60)
      COMMON /B/ETA,ETB,ETC,KA,KB,KC,WAVNO
      COMPLEX IHAT(500),ETA,ETB,ETC,KA,KB,KC,RXK,RYK,DCK
      DIMENSION PHI(100),PIN(100),NU(3)
      DO 1 I=1,NN
      Y(I)=0.
    1 CONTINUE
      NU1=NU(1)
      NU2=NU(2)
      NU3=NU(3)
      ND2=ND(2)-1
      DO 2 I=1,ND2
      RXK(I)=RCX(I)*KA
      RYK(I)=RCY(I)*KA
      DCK(I)=DC(I)*KA
    2 CONTINUE
      ETK=ETA*KA
      CALL TEEXC(IHAT,NU1,NEX,ETA,KA,WAVNO,PHI,PIN,ND)
      N2=NU1+NU2
      NR=N2+NU3
      DO 3 IM=1,ND2
      Y(IM)=P1(IM,1)/ETK
    3 CONTINUE
      CALL SYM(1,Y,1,NU1,NR,NN)
      ND3=ND(3)
      ND4=ND(4)-1
      DO 4 I=ND3,ND4
      RXK(I)=RCX(I)*KC
      RYK(I)=RCY(I)*KC
      DCK(I)=DC(I)*KC
    4 CONTINUE
      ETK=ETC*KC
      KR=(NR+1)*NU1
      KI=1
```

```
      DO 5 IM=ND3,ND4
      Y(IM-ND3+1+KR)=P1(IM,ND3)/ETK
      YHS(KI)=Y(IM-ND3+1+KR)
      KI=KI+1
5     CONTINUE
      CALL SYM(1,Y,KR+1,NU2,NR,NN)
      CALL P2Q2(YD1,NU,ND,1)
      KR=N2*NR
      K=1
      DO 6 IN=1,NU3
      KK=KR+(IN-1)*NR
      DO 6 IM=1,N2
      Y(KK+IM)=YD1(K)
      K=K+1
6     CONTINUE
      ND5=ND(5)-1
      DO 7 I=1,ND5
      RXK(I)=RCX(I)*KB
      RYK(I)=RCY(I)*KB
      DCK(I)=DC(I)*KB
7     CONTINUE
      ETK=KB/ETB
      DO 8 IN=1,ND2
      K=N2+(IN-1)*NR
      DO 8 IM=1,ND5
      Y(K+IM)=P5(IM,IN)/KB
8     CONTINUE
      KR=NU1*NR+N2
      DO 9 IN=ND3,ND4
      K=KR+(IN-ND3)*NR
      DO 9 IM=1,ND5
      Y(K+IM)=P5(IM,IN)/KB
9     CONTINUE
      KR=N2*(NR+1)
      K=KR
      DO 11 IN=1,ND5
      DO 10 IM=IN,ND5
      Y(K+IM)=(P6S(IM-1,IN-1,1,1)+P6S(IM-1,IN,1,-1)
     1+P6S(IM,IN-1,-1,1)+P6S(IM,IN,-1,-1))/ETK
10    CONTINUE
      K=K+NR
11    CONTINUE
      CALL SYM(2,Y,KR+1,NU3,NR,NN)
      RETURN
      END
```

```
      SUBROUTINE TMMAT(Y,NU,NN,IHAT,NEX,PHI,PIN,YHS)
      COMPLEX Y(NN),YD1(1000),P1,Q5,P6S,YHS(30),ETK
      COMMON /CUV/ULX(60),ULY(60),ND(5)/C/RCX(60),RCY(60),DC(60)
      COMMON /CK/RXK(60),RYK(60),DCK(60)
      COMMON /B/ETA,ETB,ETC,KA,KB,KC ,WAVNO
      COMPLEX IHAT(500),ETA,ETB,ETC,KA,KB,KC,RXK,RYK,DCK,SUM
      DIMENSION PHI(100),PIN(100),NU(3)
      DO 1 I=1,NN
      Y(I)=0.
1     CONTINUE
      NU1=NU(1)
      NU2=NU(2)
      NU3=NU(3)
      ND2=ND(2)-1
      DO 2 I=1,ND2
      RXK(I)=RCX(I)*KA
      RYK(I)=RCY(I)*KA
      DCK(I)=DC(I)*KA
2     CONTINUE
      ETK=ETA*KA
      CALL TMEXC(IHAT,NU1,NEX,ETA,KA,WAVNO,PHI,PIN,ND)
      N2=NU1+NU2
      NR=N2+NU3
      IN=2
      DO 3 IM=2,ND2
      Y(IM-1)=(P6S(IM-1,IN-1,1,1)+P6S(IM-1,IN,1,-1)
     1+P6S(IM,IN-1,-1,1)+P6S(IM,IN,-1,-1))/ETK
3     CONTINUE
      CALL SYM(1,Y,1,NU1,NR,NN)
      ND3=ND(3)
      ND31=ND3+1
      ND4=ND(4)-1
      DO 4 I=ND3,ND4
      RXK(I)=RCX(I)*KC
      RYK(I)=RCY(I)*KC
      DCK(I)=DC(I)*KC
4     CONTINUE
      ETK=ETC*KC
      KR=(NR+1)*NU1
      IN=ND31
      KI=1
      DO 5 IM=ND31,ND4
      Y(IM-ND31+1+KR)=(P6S(IM-1,IN-1,1,1)+P6S(IM-1,IN,1,-1)
     1+P6S(IM,IN-1,-1,1)+P6S(IM,IN,-1,-1))/ETK
      YHS(KI)=Y(IM-ND31+1+KR)
      KI=KI+1
5     CONTINUE
      CALL SYM(1,Y,KR+1,NU2,NR,NN)
      CALL P2Q2(YD1,NU,ND,2)
      KR=N2*NR
      K=1
```

```
      DO 6 IN=1,NU3
      KK=KR+(IN-1)*NR
      DO 6 IM=1,N2
      Y(KK+IM)=YD1(K)
      K=K+1
6     CONTINUE
      ND5=ND(5)-1
      DO 7 I=1,ND5
      RXK(I)=RCX(I)*KB
      RYK(I)=RCY(I)*KB
      DCK(I)=DC(I)*KB
7     CONTINUE
      DO 8 IN=2,ND2
      K=N2+(IN-2)*NR
      DO 8 IM=1,ND5
      Y(K+IM)=Q5(IM,IN)/KB
8     CONTINUE
      KR=NU1*NR+N2
      DO 9 IN=ND31,ND4
      K=KR+(IN-ND31)*NR
      DO 9 IM=1,ND5
      Y(K+IM)=Q5(IM,IN)/KB
9     CONTINUE
      ETK=KB/ETB
      KR=N2*(NR+1)
      K=KR
      DO 11 IN=1,ND5
      DO 10 IM=IN,ND5
      Y(K+IM)=P1(IM,IN)/ETK
10    CONTINUE
      K=K+NR
11    CONTINUE
      CALL SYM(2,Y,KR+1,NU3,NR,NN)
      RETURN
      END
```

2.8 Subroutine PCK

This subroutine computes the time average power flowing through Γ_1 given by

$$P_{t1} = \mathrm{Re} \int_{\Gamma_1} \underline{E}^b \times \underline{H}^{b*} \cdot \hat{\underline{x}}\, dy$$

and the time average power flowing through Γ_3 given by

$$P_{t3} = \mathrm{Re} \int_{\Gamma_3} \underline{E}^b \times \underline{H}^{b*} \cdot \hat{\underline{x}}\, dy$$

The above are computed by using the vectors $\vec{V}^1$, $\vec{V}^3$, and $\eta_o \vec{I}$ hence the Fortran variables PT1 and PT3 are defined by

$$PT1 = \eta_o k_o \, P_{t1}$$

$$PT2 = \eta_o k_o \, P_{t3}$$

The argument parameters are defined by

Input: X = array containing solution vector of Eq. (2-17) for one excitation.

NU = array containing number of elements in vectors $\vec{V}^1$, $\vec{V}^3$, and $\eta_o \vec{I}$ respectively.

ND = node array defined in section 2.1.1.

ID = integer variable, ID = 1 for TE case and ID = 2 for TM case.

Output: PT1 and PT2 are printed out at execution.

The main purpose of this subroutine is to perform a consistency check on the numerical solution. That is, the real power flowing across each aperture face should be the same if the material filling the slit is lossless. This computation becomes unreliable in the TM case when the width of the slit is small compared to a wavelength.

```
      SUBROUTINE PCK(X,NU,ND,ID)
      COMPLEX X(50),PT,CONJG
      COMMON /C/RCX(60),RCY(60),DC(60)
      INTEGER NU(3),ND(5)
C---------ID = 1... TE CASE
C---------ID = 2... TM CASE
      NU1=NU(1)
      NU2=NU(2)
      N2=NU1+NU2
      PT=0.
      IF(ID.EQ.1) GO TO 3
      DO 1 I=1,NU1
      PT=PT+X(I)*(DC(I)*CONJG(X(N2+I))+DC(I+1)*CONJG(X(N2+I+1)))/2.
1     CONTINUE
      PT1=-REAL(PT)
      WRITE(3,100) PT1
      N3=ND(3)-1
      N2=N2+N3
      PT=0.
      DO 2 I=1,NU2
      PT=PT+X(I+NU1)*(DC(I+N3)*CONJG(X(N2+I))
     1+DC(I+N3+1)*CONJG(X(N2+I+1)))/2.
2     CONTINUE
      PT2=REAL(PT)
      WRITE(3,101) PT2
      RETURN
3     DO 4 I=1,NU1
      PT=PT+X(I)*(CONJG(X(I+N2))+CONJG(X(I+N2+1)))*DC(I)/2.
4     CONTINUE
      PT1=REAL(PT)
      WRITE(3,100) PT1
      N3=ND(3)-1
      N2=N2+N3
      PT=0.
      DO 5 I=1,NU2
      PT=PT+X(I+NU1)*(CONJG(X(I+N2))+CONJG(X(I+N2+1)))*DC(I+N3)/2.
5     CONTINUE
      PT2=-REAL(PT)
      WRITE(3,101) PT2
100   FORMAT(' ','PTA =',2E17.7)
101   FORMAT(' ','PTC =',2E17.7)
      RETURN
      END
```

2.9 Subroutine TRANS1

This subroutine computes the transmission coefficient defined by Eq. (5-13) for each excitation. The argument parameters are defined by

Input: YHS = array containing elements of Y^c stored columnwise.

PIN = array containing $\eta_o k_o P_{iN}$ for each excitation.

PHI = excitation array defined in Section 2.1.1.

NEX = number of excitations.

VM = array containing solution vectors of Eq. (2-17) for all excitations, stored columnwise.

ID = integer variable, ID = 1 for TE case and ID = 2 for TM case.

NU = array containing number of elements in $\vec{V}^1$, $\vec{V}^3$, and $\eta_o \vec{I}$ respectively.

Output: PT = array containing time average power transmitted through aperture face Γ_3 times $\eta_o k_o$ for each excitation.

DO loops 1-6 perform the matrix multiplication necessary in Eq. (5-16). The resulting transmitted power of the jth excitation is

$$PT(j) = \eta_o k_o \; P_{t3}$$

for j = 1,2,..., NEX which is returned in array PT for use in subroutine GAIN1. The transmission coefficient is computed by Eq. (5-13) using P_{iN} computed in subroutine TMEXC or TEEXC.

```
      SUBROUTINE TRANS1(YHS,PIN,PHI,NEX,VM,ID,PT,NU)
      COMPLEX YHS(30),YAUX(30),VC(30),VM(1000)
      COMPLEX CONJG,SUM,S
      DIMENSION PHI(100),PIN(100),PT(100),NU(3)
      DATA PI/3.141593/
100    FORMAT('-','EXCITATIONS AND TRANSMISSION COEFFS.')
101   FORMAT(' ',3E11.4)
      NTU=NU(1)+NU(2)+NU(3)
      NSK=NU(1)
      NU2=NU(2)
      DO 1 I=1,NU2
      YAUX(I)=2.*CONJG(YHS(I))
1     CONTINUE
      WRITE(3,100)
      DO 10 J=1,NEX
      DO 3 I=1,NU2
      VC(I)=CONJG(VM(I+NSK))
3     CONTINUE
      S=0.
      DO 4 I=1,NU2
      S=S+VM(I+NSK)*VC(I)
4     CONTINUE
      SUM=YAUX(1)*S
      DO 5 L=2,NU2
      S=0.
      NLI=NU2-L+1
      DO 6 I=1,NLI
      S=S+VM(I+NSK)*VC(L+I-1)+VM(NSK+L+I-1)*VC(I)
6     CONTINUE
      SUM=SUM+YAUX(L)*S
5     CONTINUE
      PT(J)=REAL(SUM)
      T=PT(J)/PIN(J)
      WRITE(3,101) PHI(2*J-1),PHI(2*J),T
      NSK=NSK+NTU
10    CONTINUE
      RETURN
      END
```

2.10 Subroutine GAIN1

This subroutine computes a gain and normalized far field pattern from Eqs. (5-20) to (5-23). The argument parameters are defined by:

Input: XI = array containing solution vectors of (2-17) for each excitation.

PHI = excitation array defined in Section 2.1.1.

DPHI = *increment*, in degrees, at which field patterns are computed.

NI = number of points in region c which far field is computed to obtain patterns.

PT = $k_o \eta_o P_{t3}$ computed for each excitation in subroutine TRANS1.

ID = integer option variable, ID = 1 for TE case and ID = 2 for TM case.

NEX = number of excitations.

NU = array containing number of elements in $\vec{V}^1$, $\vec{V}^3$, and $\eta_o \vec{I}$ respectively.

Output: The gain and field patterns are printed out.

The field is computed in region c by the use of a measurement vector defined in Section 5.1 of [1]. Here the reference frame is chosen so that the origin is at the center of Γ_3 and the y axis is coincident with Γ_3. The variable YOC is the center of Γ_3 times k_c relative to the old coordinates of Fig. 1. DO loop 2 forms the measurement vector and DO loop 10 computes the patterns for each excitation.

```
      SUBROUTINE GAIN1(XI,PHI,DPHI,NI,PT,ID,NEX,NU)
      COMMON /CUV/ULX(60),ULY(60),ND(5)/C/RCX(60),RCY(60),DC(60)
      COMMON /B/ETA,ETB,ETC,KA,KB,KC,WAVNO
      COMPLEX FT(1500),XI(1000),U,CEXP,S,S1,S2,CSIN,HN1,HN2,YOC
      COMPLEX ETA,ETB,ETC,KA,KB,KC,DCN1,DCN2,DCK(30),RYK(30),CONST
      DIMENSION PHI(100),FP(100),GA(100),PT(100),NU(3)
      DATA PH0/-1.570796/,U/(0.,1.)/
100   FORMAT('1','TE GAIN AND FIELD PATTERNS')
101   FORMAT('1','TM GAIN AND FIELD PATTERNS')
102   FORMAT(' ',3E11.4)
      ND3=ND(3)
      ND4=ND(4)
      DPH=-DPHI*PH0/90.
      NTU=NU(1)+NU(2)+NU(3)
      DO 1 I=ND3,ND4
      RYK(I)=KC*RCY(I)
      DCK(I)=KC*DC(I)
1     CONTINUE
      K=1
      YOC=KC*(RCY(ND3)+DC(ND3)/2.+RCY(ND4-1)-DC(ND4-1)/2.)/2.
      N4=ND4-ID
      PH=PH0
      DO 2 I=1,NI
      SN=SIN(PH)
      CPH=COS(PH)
      DO 3 J=ND3,N4
      HN1=U*(RYK(J)-YOC)
      HN2=U*(RYK(J+1)-YOC)
      DCN1=DCK(J)/2.
      DCN2=DCK(J+1)/2.
      IF(ID.NE.1) GO TO 5
      IF(PH.EQ.0.) GO TO 6
      FT(K)=-DCK(J)*CEXP(HN1*SN)*CSIN(DCN1*SN)/(DCN1*SN)
      K=K+1
      GO TO 8
6     FT(K)=-DCK(J)
      K=K+1
      GO TO 8
5     IF(PH.EQ.0.) GO TO 7
      S1=CEXP(HN2*SN)*CSIN(DCN2*SN)/(DCN2*SN)**2
      S2=CEXP(HN1*SN)*CSIN(DCN1*SN)/(DCN1*SN)**2
      FT(K)=CPH/(2.*U)*(DCK(J+1)*S1-DCK(J)*S2)
      K=K+1
      GO TO 8
7     FT(K)=-(DCK(J)+DCK(J+1))/2.
      K=K+1
8     CONTINUE
3     CONTINUE
      PH=PH+DPH
2     CONTINUE
```

```
      NU2=NU(2)
      NSK=NU(1)
      DO 10 J=1,NEX
      K=1
      DO 12 I=1,NI
      S=0.
      DO 11 L=1,NU2
      S=S+XI(NSK+L)*FT(K)
      K=K+1
11    CONTINUE
      F=CABS(S)
      FP(I)=F
      GA(I)=F*F/(2.*KC*ETC*PT(J))
12    CONTINUE
      FM=FP(1)
      DO 13 I=2,NI
      IF(FM.LT.FP(I)) FM=FP(I)
13    CONTINUE
      DO 14 I=1,NI
      FP(I)=FP(I)/FM
14    CONTINUE
      PH=-90.
      IF(ID.EQ.1) WRITE(3,100)
      IF(ID.EQ.2) WRITE(3,101)
      DO 15 I=1,NI
      WRITE(3,102) PH,GA(I),FP(I)
      PH=PH+DPHI
15    CONTINUE
      NSK=NSK+NTU
10    CONTINUE
      RETURN
      END
```

2.11 Subroutine SYM

This subroutine is used when only a portion of a submatrix of a larger matrix Y is computed and needs to be stored appropriately in the larger matrix. It is applicable when the submatrix is symmetric or symmetric Toeplitz. The matrix Y has dimension IR × IR and is stored columnwise. The argument parameters are defined by

Input: ID = integer option variable, ID = 1 for a symmetric Toeplitz matrix and ID = 2 for a symmetric matrix.

Y = larger matrix stored columnwise.

K1 = position of the first element of the first column of the NR × NR submatrix in Y.

NR = number of rows of submatrix.

IR = number of rows of matrix Y.

NN = IR × IR

If ID = 1, the first column of the submatrix is computed and stored in Y. If ID = 2, the lower triangular portion of the submatrix is computed and stored in Y. These operations are done before SYM is called. After calling SYM, the array Y is returned with the appropriate locations filed in utilizing the special property of the submatrix.

```
      SUBROUTINE SYM(ID,Y,K1,NR,IR,NN)
      COMPLEX Y(NN)
      IF(ID.NE.1) GO TO 5
      DO 2 J=2,NR
      K=K1+(J-1)*IR
      DO 1 I=2,J
      Y(K)=Y(K1+1+J-I)
      K=K+1
1     CONTINUE
      JN=NR-J+1
      DO 3 I=1,JN
      Y(K)=Y(K1+I-1)
      K=K+1
3     CONTINUE
2     CONTINUE
      RETURN
 5    L1=K1
      DO 6 J=2,NR
      L2=L1+IR
      JN=NR-J+1
      DO 7 I=1,JN
      Y(L2)=Y(L1+I)
      L2=L2+IR
7     CONTINUE
      L1=L1+IR+1
6     CONTINUE
      RETURN
      END
```

2.12 Subroutines DECOMP and SOLVE

These subroutines are used to solve a system of N linear equations in N unknowns, specifically Eq. (2-17). The input to DECOMP consists of N, NN = N*N, and the matrix of coefficients stored columnwise in UL. The output from DECOMP is IPS and UL which is used by SOLVE. The rest of the input to SOLVE consists of N and the excitation vector stored in B. SOLVE returns the solution vector in X and must be called for every excitation. More detail concerning DECOMP and SOLVE may be found on pages 46-49 of [4].

```
      SUBROUTINE DECOMP(N,NN,IPS,UL)
      COMPLEX UL(NN),PIVOT,EM
      DIMENSION SCL(50),IPS(50)
      DO 5 I=1,N
      IPS(I)=I
      RN=0.
      J1=I
      DO 2 J=1,N
      ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
      J1=J1+N
      IF(RN-ULM) 1,2,2
    1 RN=ULM
    2 CONTINUE
      SCL(I)=1./RN
    5 CONTINUE
      NM1=N-1
      K2=0
      DO 17 K=1,NM1
      BIG=0.
      DO 11 I=K,N
      IP=IPS(I)
      IPK=IP+K2
      SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
      IF(SIZE-BIG) 11,11,10
   10 BIG=SIZE
      IPV=I
   11 CONTINUE
      IF(IPV-K) 14,15,14
   14 J=IPS(K)
```

```
      IPS(K)=IPS(IPV)
      IPS(IPV)=J
   15 KPP=IPS(K)+K2
      PIVOT=UL(KPP)
      KP1=K+1
      DO 16 I=KP1,N
      KP=KPP
      IP=IPS(I)+K2
      EM=-UL(IP)/PIVOT
   18 UL(IP)=-EM
      DO 16 J=KP1,N
      IP=IP+N
      KP=KP+N
      UL(IP)=UL(IP)+EM*UL(KP)
   16 CONTINUE
      K2=K2+N
   17 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SOLVE(N,NN,IPS,UL,B,X)
      COMPLEX UL(NN),B(N),X(N),SUM
      DIMENSION IPS(50)
      NP1=N+1
      IP=IPS(1)
      X(1)=B(IP)
      DO 2 I=2,N
      IP=IPS(I)
      IPB=IP
      IM1=I-1
      SUM=0.
      DO 1 J=1,IM1
      SUM=SUM+UL(IP)*X(J)
    1 IP=IP+N
    2 X(I)=B(IPB)-SUM
      K2=N*(N-1)
      IP=IPS(N)+K2
      X(N)=X(N)/UL(IP)
      DO 4 IBACK=2,N
      I=NP1-IBACK
      K2=K2-N
      IPI=IPS(I)+K2
      IP1=I+1
      SUM=0.
      IP=IPI
      DO 3 J=IP1,N
      IP=IP+N
    3 SUM=SUM+UL(IP)*X(J)
    4 X(I)=(X(I)-SUM)/UL(IPI)
      RETURN
      END
```

2.13 Subroutine SOLN

This subroutine forms the system of Eqs. (2-17) for the TE or TM case and solves for the unknown vectors $\vec{V}^1$, $\vec{V}^3$, and $\eta_o\vec{I}$ for each excitation. The argument parameters are defined by:

Input: IJ = integer option variable, IJ = 1 for TE case and IJ = 2 for TM case.

IJOB = integer option array defined in Section 2.1.1.

PHI = excitation array defined in Section 2.1.1.

ND = Nodal array defined in Section 2.1.1.

NEX = number of excitations to be considered.

NI = number of points in region c at which far field is to be evaluated to obtain a pattern.

DPHI = increment, in degrees, at which gain pattern is computed.

Output: The magnetic and electric currents are printed out as well as the transmission coefficient, gain pattern, and normalized field pattern if desired. This is done for each excitation.

The entire coefficient matrix of Eq. (2-17) is stored in array Y by columns. The elements of the vector $\vec{I}^i$, are stored in array IX for each excitation. Each right hand side vector of (2-17) is formed in array X, the non-zero elements of which are selected from IX. The number of unknowns is different for each polarization and are stored in array NU as

$NU(1)$ = number of elements in $\vec{V}^1$

$NU(2)$ = number of elements in $\vec{V}^3$

$NU(3)$ = number of elements in $\eta_o\vec{I}$

Subroutine SOLVE is called for each excitation and the solution vector is returned in array ANS. The magnetic and electric currents are then printed out if desired. The transmission coefficient is computed by TRANS1 and the gain and normalized field patterns are computed by GAIN1. Also PCK is called for each excitation to compute the time average power across Γ_1 and Γ_3 which serves as a consistency check on the numerical solution when applicable.

```
      SUBROUTINE SOLN(IJ,IJOB,PHI,ND,NEX,NI,DPHI)
      COMPLEX Y(2500),IX(500),ANS(50),X(50),YHS(30),VM(1000)
      INTEGER IJOB(6),IPS(50),NU(3),NC(5)
      DIMENSION PHI(100),PIN(100),PT(100)
      NU(1)=ND(2)-IJ
      NU(2)=ND(4)-ND(3)-IJ+1
      NU(3)=ND(5)-1
      NTU=NU(1)+NU(2)+NU(3)
      NN=NTU*NTU
      IF(IJ.EQ.1) CALL TEMAT(Y,NU,NN,IX,NEX,PHI,PIN,YHS)
      IF(IJ.EQ.2) CALL TMMAT(Y,NU,NN,IX,NEX,PHI,PIN,YHS)
      CALL DECOMP(NTU,NN,IPS,Y)
      NU1=NU(1)
      NU2=NU(2)+NU1
      N1=NU1+1
      N2=NU2+1
      KI=1
      DO 1 J=1,NEX
      DO 2 I=1,NU1
      X(I)=IX((J-1)*NU1+I)
2     CONTINUE
      DO 3 I=N1,NTU
      X(I)=0.
3     CONTINUE
      CALL SOLVE(NTU,NN,IPS,Y,X,ANS)
```

```
      IF(IJOB(4).NE.1) GO TO 7
      IF(IJ.EQ.1) WRITE(3,101)
      IF(IJ.EQ.2) WRITE(3,102)
      IF(PHI(2*J-1).EQ.0.) WRITE(3,104) PHI(2*J)
      IF(PHI(2*J-1).NE.0.) WRITE(3,103) PHI(2*J-1),PHI(2*J)
      WRITE(3,105)
      IF(IJ.EQ.1) WRITE(3,107)
      IF(IJ.EQ.2) WRITE(3,108)
      K=IJ
      DO 4 I=1,NU1
      CM=CABS(ANS(I))
      WRITE(3,100) K,ANS(I),CM
      K=K+1
4     CONTINUE
      K=ND(3)+IJ-1
      DO 5 I=N1,NU2
      CM=CABS(ANS(I))
      WRITE(3,100) K,ANS(I),CM
      K=K+1
5     CONTINUE
      WRITE(3,106)
      K=1
      IF(IJ.EQ.1) WRITE(3,108)
      IF(IJ.EQ.2) WRITE(3,107)
      DO 6 I=N2,NTU
      CM=CABS(ANS(I))
      WRITE(3,100) K,ANS(I),CM
      K=K+1
6     CONTINUE
7     CONTINUE
      DO 8 I=1,NTU
      VM(KI)=ANS(I)
      KI=KI+1
8     CONTINUE
      CALL PCK(ANS,NU,ND,IJ)
1     CONTINUE
      IF(IJOB(5).EQ.1) CALL TRANS1(YHS,PIN,PHI,NEX,VM,IJ,PT,NU)
107   FORMAT('-','SEGMENT NO.',7X,'REAL',13X,'IMAG.',12X,'MAG.')
108   FORMAT('-','NODE NO.',10X,'REAL',13X,'IMAG.',12X,'MAG.')
      RETURN
      END
```

Chapter 3
PROGRAM II - MODAL SOLUTION

The purpose of this computer program is to analyze the transmission properties of a filled slit whose cross section may be viewed as a chain of two-dimensional rectangular regions as treated in [1, Chapter 6]. Descriptions of how to use the program and set up the required data cards are presented first followed by detailed descriptions of all the subroutines.

3.1.1 Required Input Data and Main Program Description

The input data required by the user is read in the main program from a sequence of data cards according to the format statements:

```
100  FORMAT(6E11.4)
101  FORMAT(8I5)
102  FORMAT(2E20.7)
```

The sequence of data cards is arranged as shown in Table 4. The variables NGQ, A, T, IJOB, and PHI are defined in Section 2.1.1. The coordinates used to specify the excitation are shown in Fig. 1. For simplicity, the material filling the slit is assumed to be homogeneous and lossy if desired. The two half space regions are assumed loss-free. Thus the electrical properties are given by

$$\text{NMUA, NEPSA} = \mu_a/\mu_o,\ \varepsilon_a/\varepsilon_o$$

$$\text{NMUB, NEPSB} = \mu_b/\mu_o,\ \varepsilon_b/\varepsilon_o$$

$$\text{NMUC, NEPSC} = \mu_c/\mu_o,\ \varepsilon_c/\varepsilon_o$$

where NMUB and NEPSB are specified by a complex number x-jy for x and y

Table 4. Arrangement of data cards for Program II. N1 = NEX + NGQ + 1

Data Card Number	Format Number	Information to be Typed on Card
1	101	NGQ
2	102	A(1), T(1)
⋮	⋮	
1 + NGQ	102	A(NGQ), T(NGQ)
2 + NGQ	101	IJOB(1), IJOB(2),..., IJOB(6)
3 + NGQ	100	PHI(1), PHI(2)
⋮	⋮	⋮ ⋮
1 + N1	100	PHI(2*NEX-1), PHI(2*NEX)
2 + N1	100	NMUA, NEPSA
3 + N1	100	NMUB, NEPSB
4 + N1	100	NMUC, NEPSC
5 + N1	100	FMC, DPHI
6 + N1	101	NB, NI
7 + N1	100	W(1), W(2),..., W(NB+1)
8 + N1	100	H(1), H(2),..., H(NB)
9 + N1	100	D(1), D(2),..., D(NB)
10 + N1	100	YL(1), YL(2),..., YL(NB)
11 + N1	100	YR(1), YR(2),..., YR(NB)
12 + N1	101	N(1), N(2),..., N(NB+1)
13 + N1	101	NT(1), NT(2),..., NT(NB)

non-negative real numbers. FMC, DPHI, and NI are the same as in Section 2.1.1. The rest of the variables are defined by:

NB = number of rectangular regions comprising slit cross section.

W = array containing w_q, q = 1,2,..., NB+1

H = array containing h_q, q = 1,2,..., NB

D = array containing d_q, q = 1,2,..., NB

YL = array containing $y_{\ell q}$, q = 1,2,..., NB

YR = array containing Y_{rq}, q = 1,2,..., NB

N = array containing numbers N_q of subsections into which each aperture is divided, q = 1,2,..., NB+1

NT = array containing numbers which equal the maximum number of waveguide modes used to compute the fields in each rectangular region.

All lengths are in meters where a typical rectangular sub-region is shown in Fig. 4.

If region b is lossy, the skin depth in meters and the conductivity in siemens/meter is computed and printed out. DO loop 1 multiplies all dimensions by k_o, the wavelength of free space. The upper triangular portion of the matrix of coefficients in Eq. (6-8) and the elements of $\vec{I}^i$ are formed by calling subroutine MAT. The result is returned to the main program in arrays Y and XI respectively, stored columnwise. The symmetric system of equations is solved by calling subroutine GELS. The transmission coefficient is computed in subroutine TRANS2 and the gain and normalized field patterns are computed in GAIN2.

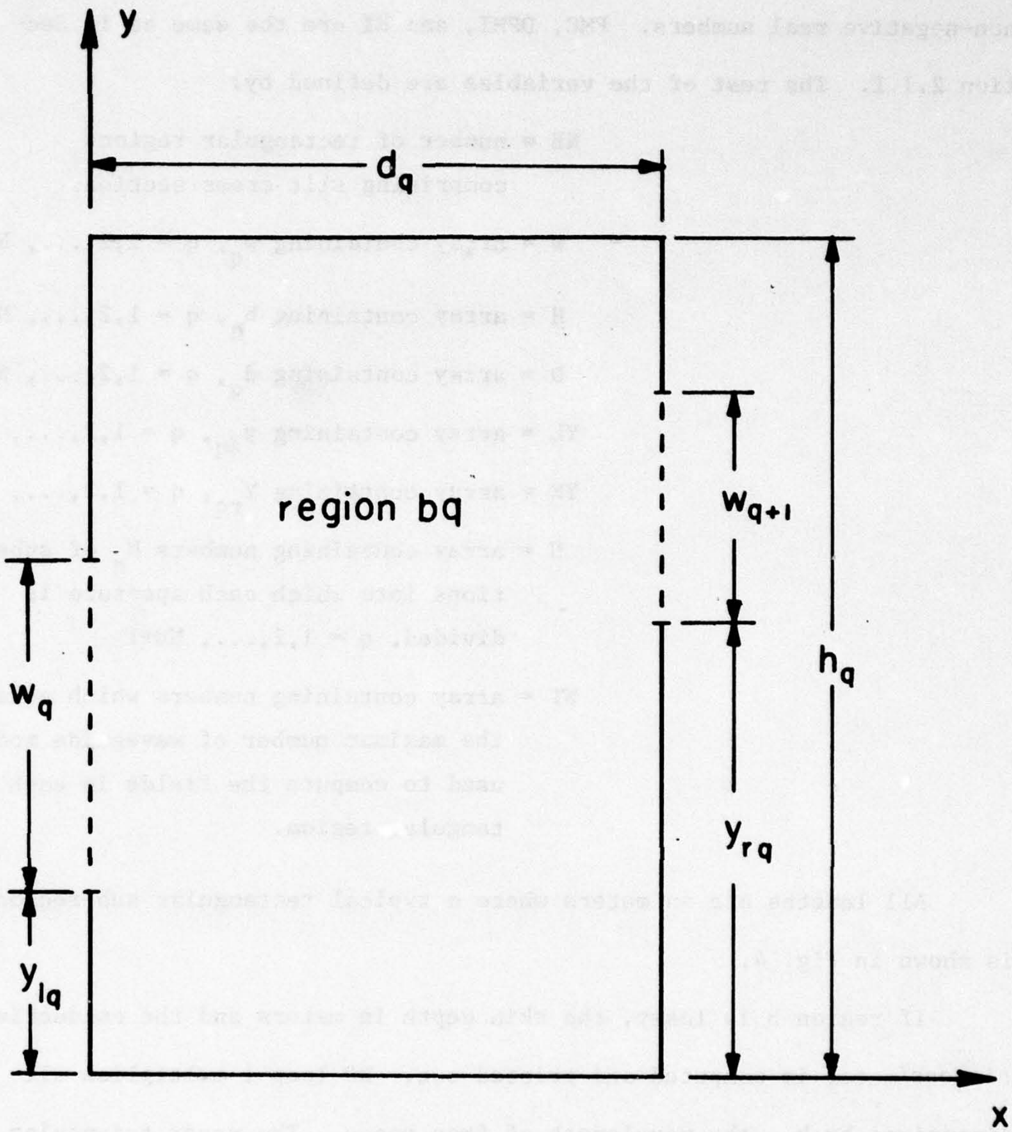

Fig. 4. Dimensions of the qth rectangular sub-region of region b.

3.1.2 Listing of Main Program and Sample Output

```
C
C---------THICK SLIT PROGRAM TWO...
C         MOCAL SOLUTION FOR MULTIPLE FEGION FILLED SLIT
C         IN A GRCUNO PLANE OF FINITE THICKNESS
C
      COMPLEX Y(4000),XI(5000),YHSC(40)
      COMPLEX CSQRT,ETANB,NMUB,NEPSB,WAVNB
      COMPLEX YHS(40),XZ(40),YZ(40),C1
      DIMENSION PHI(100),PIN(100),IJOB(6),NU(10),PT(100)
      REAL NMUA,NMUC,NEPSA,NEPSC
      COMMON /GQI/A(10),T(10),NGQ
      COMMON /E/ETANA,ETANB,ETANC,WAVNA,WAVNB,WAVNC,WAVNO
      COMMON /B/W(10),H(10),YL(10),YR(10),DC(10),D(10),N(10),NT(10),NB
      DATA PI/3.141593/
100   FORMAT(6E11.4)
101   FORMAT(8I5)
102   FORMAT(2E20.7)
      READ(1,101) NGQ
      READ(1,102)(A(I),T(I),I=1,NGQ)
      WRITE(3,101) NGQ
      WRITE(3,102)(A(I),T(I),I=1,NGQ)
C---------IJCB(1) = 1... TE CASE
C         IJCB(2) =1...TM CASE
C         IJCB(3) = NC. CF EXCITATICNS
C         IJCB(4) = 1...PRINT OUT CURRENTS
C         IJCB(5) = 1...CCMP. TRANSMISSICN CCEFFS.
C         IJCB(6) = 1...CCMP. GAIN ANC FIELC PATTERNS
      READ(1,101)(IJCB(I),I=1,6)
      NEX=IJOB(3)
      WRITE(3,101)(IJOB(I),I=1,6)
      K=1
      DO 3 I=1,NEX
      READ(1,100) PHI(K),PHI(K+1)
      WRITE(3,100) PHI(K),PHI(K+1)
      K=K+2
3     CONTINUE
      READ(1,100) NMUA,NEPSA
C         THE MATERIAL FILLING THE SLIT IS ASSUMED HOMCGENEOUS
      READ(1,100) NMUB,NEPSB
      READ(1,100) NMUC,NEPSC
      READ(1,100) FNC,DPHI
      READ(1,101) NB,NI
      NB1=NB+1
      READ(1,100)(W(I),I=1,NB1)
      READ(1,100)(H(I),I=1,NB)
      READ(1,100)(C(I),I=1,NB)
      READ(1,100)(YL(I),I=1,NB)
      READ(1,100)(YR(I),I=1,NB)
      READ(1,101)(N(I),I=1,NB1)
      READ(1,101)(NT(I),I=1,NB)
```

```
      WAVNO=PI*FMC/150.
      WAVNA=SQRT(NMUA*NEPSA)
      WAVNB=CSQRT(NMUB*NEPSB)
      WAVNC=SQRT(NMUC*NEPSC)
      ETANA=SQRT(NMUA/NEPSA)
      ETANB=CSQRT(NMUB/NEPSB)
      ETANC=SQRT(NMUC/NEPSC)
      IF(AIMAG(NEPSB).EQ.0.) GO TO 2
      SIG=-FMC*AIMAG(NEPSB)/.18E 5
      SKD=-1./(WAVNO*AIMAG(WAVNB))
      WRITE(3,215) SIG
      WRITE(3,216) SKD
2     CONTINUE
200   FORMAT('1',15X,'THICK SLIT PARAMETERS...')
203   FORMAT('-',15X,'REGION PARAMETERS NORMALIZED BY THOSE OF',
     11X,'FREE SPACE')
204   FORMAT('-',15X,'REGION A',14X,'REGION C',14X,'REGIONB')
206   FORMAT('-','PERMEABILITY',3X,E11.4,11X,E11.4,11X,2E11.4)
207   FORMAT('-','PERMITTIVITY',3X,E11.4,11X,E11.4,11X,2E11.4)
208   FORMAT('-','IMPEDANCE',6X,E11.4,11X,E11.4,11X,2E11.4)
209   FORMAT('-','WAVENUMBER',5X,E11.4,11X,E11.4,11X,2E11.4)
210   FORMAT('-',15X,'NO. OF RECTANGULAR REGIONS =',I5)
211   FORMAT('-',15X,'REGION B INDEX',I3)
212   FORMAT(' ',20X,'W =',E11.4,5X,'N =',I5)
213   FORMAT(' ',20X,'H =',E11.4,5X,'D =',E11.4)
214   FORMAT(' ',20X,'YL =',E11.4,4X,'YR =',E11.4,5X,'NT =',I5)
215   FORMAT('-','CONDUCTIVITY OF REGION B =',E11.4,2X,'MHOS/METER')
216   FORMAT('-','SKIN DEPTH OF REGION B =',E11.4,2X,'METERS')
      WRITE(3,200)
      WRITE(3,210) NB
      DO 6 I=1,NB
      WRITE(3,211) I
      WRITE(3,212) W(I),N(I)
      WRITE(3,213) H(I),D(I)
      WRITE(3,214) YL(I),YR(I),NT(I)
6     CONTINUE
      WRITE(3,211) NB1
      WRITE(3,212) W(NB1),N(NB1)
      WRITE(3,203)
      WRITE(3,204)
      WRITE(3,206) NMUA,NMUC,NMUB
      WRITE(3,207) NEPSA,NEPSC,NEPSB
      WRITE(3,208) ETANA,ETANC,ETANB
      WRITE(3,209) WAVNA,WAVNC,WAVNB
      DO 1 I=1,NB
      W(I)=W(I)*WAVNO
      H(I)=H(I)*WAVNO
      DC(I)=W(I)/N(I)
      D(I)=D(I)*WAVNO
      YL(I)=YL(I)*WAVNO
      YR(I)=YR(I)*WAVNO
1     CONTINUE
      W(NB1)=W(NB1)*WAVNO
      DC(NB1)=W(NB1)/N(NB1)
      IF(IJOB(1).NE.1) GO TO 10
```

```
C--------COMPUTE MATRIX FOR TE CASE...IJOB(1)=1
      CALL MAT(Y,NR,1,XI,YHSC,NEX,PHI,NU,PIN)
107   FORMAT('-',1X,'POS.',6X,'REAL',13X,'IMAG',13X,'MAG')
      MR=NR*(NR+1)/2
      CALL GELS(XI,Y,NR,NEX,MR)
      IF(IJOB(4).NE.1) GO TO 8
105   FORMAT('-',10X,'APERTURE FACE',I5,3X,'...',//)
104   FORMAT(' ',I5,4E17.7)
103   FORMAT('1',15X,'TE MAGNETIC CURRENTS FOR PHI =',2E11.4)
      K=1
      KC=1
      DO 7 J=1,NEX
      WRITE(3,103) PHI(KC),PHI(KC+1)
      WRITE(3,107)
      DO 4 I=1,NB1
      WRITE(3,105) I
      N1=NU(I)
      DO 4 II=1,N1
      VA=CABS(XI(K))
      WRITE(3,104) II,XI(K),VA
      K=K+1
4     CONTINUE
      KC=KC+2
7     CONTINUE
8     CONTINUE
C------- COMP. TE TRANS. COEFFS. AND GAIN PATTERNS
      IF(IJOB(5).EQ.1) CALL TRANS2(YHSC,PIN,PHI,NEX,XI,1,PT)
      IF(IJOB(6).EQ.1) CALL GAIN2(XI,PHI,DPHI,NI,PT,1,NEX)
10    IF(IJOB(2).NE.1) GO TO 20
C---------COMPUTE MATRIX FOR TM CASE...IJOB(2)=1
      CALL MAT(Y,NR,2,XI,YHSC,NEX,PHI,NU,PIN)
      MR=NR*(NR+1)/2
      CALL GELS(XI,Y,NR,NEX,MR)
      IF(IJOB(4).NE.1) GO TO 9
303   FORMAT('1','TM MAGNETIC CURRENTS FOR PHI =',2E11.4)
      K=1
      KC=1
      DO 17 J=1,NEX
      WRITE(3,303) PHI(KC),PHI(KC+1)
      WRITE(3,107)
      DO 14 I=1,NB1
      WRITE(3,105) I
      N1=NU(I)
      DO 14 II=1,N1
      VA=CABS(XI(K))
      WRITE(3,104)II,XI(K),VA
      K=K+1
14    CONTINUE
      KC=KC+2
17    CONTINUE
9     CONTINUE
C---------COMP. TM TRANS COEFFS. AND GAIN PATTERNS
      IF(IJOB(5).EQ.1) CALL TRANS2(YHSC,PIN,PHI,NEX,XI,2,PT)
      IF(IJOB(6).EQ.1) CALL GAIN2(XI,PHI,DPHI,NI,PT,2,NEX)
20    CONTINUE
      STOP
      END
```

Table 5. Arrangement of data cards to produce sample output in Program II. $ signs are in column one.

```
$DATA
   4
.1739274E 0             -.4305682E 0
.1739274E 0             .4305682E 0
.3260725E 0             -.1699905E 0
.3260725E 0             .1699905E 0
   1     1     2     1     1     1
0.E 0       0.E 0
0.E 0       45.E 0
1.E 0        1.E 0
1.E 0       0.E 0        1.E 0       0.E 0
1.E 0       1.E 0
300.E 0      2.E 0
   2     91
.499E 0      .2E 0       .2E 0
.499E 0      .2E 0
.1E 0       .1E 0
0.E 0       0.E 0
.1E 0       0.E 0
   5     4     4
  50    50
$STOP
/*
```

Sample output is presented for the slit in Fig. 1 where the data cards are shown in Table 5. Only the $\phi^i = 0$ results are shown. The dimension $w_1 = h_1 = 0.499$ is used here instead of 0.5 as indicated from Fig. 1 to avoid numerical problems arising from the resonant dimensions of rectangular region b1.

```
            4
            0.1739274E 00      -0.4305682E 00
            0.1739274E 00       0.4305682E 00
            0.3260725E 00      -0.1699905E 00
            0.3260725E 00       0.1699905E 00
         1     1     2     1     1     1
      0.0000E 00 0.0000E 00
      0.0000E 00 0.4500E 02


THICK SLIT PARAMETERS...

NO. OF RECTANGULAR REGIONS =     2


REGION B  INDEX  1
     W = 0.4990E 00       N =     5
     H = 0.4990E 00       D = 0.1000E 00
    YL = 0.0000E 00      YR = 0.1000E 00       NT =   50


REGION B  INDEX  2
     W = 0.2000E 00       N =     4
     H = 0.2000E 00       D = 0.1000E 00
    YL = 0.0000E 00      YR = 0.0000E 00       NT =   50


REGION B  INDEX  3
     W = 0.2000E 00       N =     4


REGION PARAMETERS NORMALIZED BY THOSE OF FREE SPACE


                  REGION A           REGION C             REGIONB

PERMEABILITY      0.1000E 01         0.1000E 01           0.1000E 01 0.0000E 00

PERMITTIVITY      0.1000E 01         0.1000E 01           0.1000E 01 0.0000E 00

IMPEDANCE         0.1000E 01         0.1000E 01           0.1000E 01 0.0000E 00

WAVENUMBER        0.1000E 01         0.1000E 01           0.1000E 01 0.0000E 00
```

TE MAGNETIC CURRENTS FOR PHI = 0.0000E 00 0.0000E 00

POS.	REAL	IMAG	MAG
	APERTURE FACE	1 ...	
1	0.9809859E 00	0.5196739E 00	0.1110132E 01
2	0.6058290E 00	0.4234206E 00	0.7391304E 00
3	0.6087545E 00	0.4619285E 00	0.7641727E 00
4	0.5193210E 00	0.6850150E 00	0.8596160E 00
5	0.8455284E 00	0.8714415E 00	0.1214219E 01
	APERTURE FACE	2 ...	
1	0.4048090E 00	-0.1018196E 01	0.1095716E 01
2	0.1796385E 00	-0.7224578E 00	0.7444564E 00
3	0.1511654E 00	-0.7370691E 00	0.7524107E 00
4	0.2256975E 00	-0.1206456E 01	0.1227386E 01
	APERTURE FACE	3 ...	
1	-0.6475391E 00	-0.1721290E 01	0.1839061E 01
2	-0.3358417E 00	-0.1164394E 01	0.1211859E 01
3	-0.3503965E 00	-0.1179538E 01	0.1230482E 01
4	-0.6894223E 00	-0.1768572E 01	0.1898196E 01

EXCITATIONS AND TRANSMISSION COEFFS.

0.0000E 00 0.0000E 00 0.5564E 00
0.0000E 00 0.4500E 02 0.4952E 00

TE GAIN AND FIELD PATTERNS

-0.9000E 02	0.9197E 00	0.9234E 00
-0.8800E 02	0.9199E 00	0.9235E 00
-0.8600E 02	0.9204E 00	0.9238E 00
-0.8400E 02	0.9213E 00	0.9242E 00
-0.8200E 02	0.9226E 00	0.9248E 00
-0.8000E 02	0.9242E 00	0.9256E 00
-0.7800E 02	0.9261E 00	0.9266E 00
-0.7600E 02	0.9284E 00	0.9277E 00
-0.7400E 02	0.9310E 00	0.9290E 00
-0.7200E 02	0.9339E 00	0.9305E 00
-0.7000E 02	0.9371E 00	0.9321E 00
-0.6800E 02	0.9406E 00	0.9338E 00
-0.6600E 02	0.9444E 00	0.9357E 00
-0.6400E 02	0.9484E 00	0.9377E 00
-0.6200E 02	0.9527E 00	0.9398E 00
-0.6000E 02	0.9572E 00	0.9420E 00

-0.5800E 02	0.9619E 00	0.9443E 00
-0.5600E 02	0.9668E 00	0.9467E 00
-0.5400E 02	0.9718E 00	0.9492E 00
-0.5200E 02	0.9770E 00	0.9517E 00
-0.5000E 02	0.9823E 00	0.9543E 00
-0.4800E 02	0.9877E 00	0.9569E 00
-0.4600E 02	0.9931E 00	0.9596E 00
-0.4400E 02	0.9986E 00	0.9622E 00
-0.4200E 02	0.1004E 01	0.9649E 00
-0.4000E 02	0.1010E 01	0.9675E 00
-0.3800E 02	0.1015E 01	0.9701E 00
-0.3600E 02	0.1020E 01	0.9727E 00
-0.3400E 02	0.1026E 01	0.9752E 00
-0.3200E 02	0.1031E 01	0.9777E 00
-0.3000E 02	0.1036E 01	0.9800E 00
-0.2800E 02	0.1041E 01	0.9823E 00
-0.2600E 02	0.1045E 01	0.9845E 00
-0.2400E 02	0.1050E 01	0.9866E 00
-0.2200E 02	0.1054E 01	0.9886E 00
-0.2000E 02	0.1058E 01	0.9904E 00
-0.1800E 02	0.1062E 01	0.9921E 00
-0.1600E 02	0.1065E 01	0.9937E 00
-0.1400E 02	0.1068E 01	0.9951E 00
-0.1200E 02	0.1071E 01	0.9963E 00
-0.1000E 02	0.1073E 01	0.9973E 00
-0.8000E 01	0.1075E 01	0.9982E 00
-0.6000E 01	0.1076E 01	0.9990E 00
-0.4000E 01	0.1078E 01	0.9995E 00
-0.2000E 01	0.1078E 01	0.9998E 00
0.0000E 00	0.1079E 01	0.1000E 01
0.2000E 01	0.1079E 01	0.1000E 01
0.4000E 01	0.1078E 01	0.9998E 00
0.6000E 01	0.1077E 01	0.9994E 00
0.8000E 01	0.1076E 01	0.9988E 00
0.1000E 02	0.1074E 01	0.9981E 00
0.1200E 02	0.1072E 01	0.9971E 00
0.1400E 02	0.1070E 01	0.9960E 00
0.1600E 02	0.1067E 01	0.9948E 00
0.1800E 02	0.1064E 01	0.9934E 00
0.2000E 02	0.1061E 01	0.9918E 00
0.2200E 02	0.1057E 01	0.9901E 00
0.2400E 02	0.1053E 01	0.9883E 00
0.2600E 02	0.1049E 01	0.9863E 00
0.2800E 02	0.1045E 01	0.9843E 00
0.3000E 02	0.1040E 01	0.9821E 00
0.3200E 02	0.1036E 01	0.9798E 00
0.3400E 02	0.1031E 01	0.9775E 00
0.3600E 02	0.1026E 01	0.9751E 00
0.3800E 02	0.1020E 01	0.9726E 00
0.4000E 02	0.1015E 01	0.9701E 00
0.4200E 02	0.1010E 01	0.9676E 00
0.4400E 02	0.1004E 01	0.9650E 00
0.4600E 02	0.9992E 00	0.9625E 00
0.4800E 02	0.9939E 00	0.9599E 00
0.5000E 02	0.9887E 00	0.9574E 00

0.5200E 02	0.9835E 00	0.9549E 00
0.5400E 02	0.9785E 00	0.9525E 00
0.5600E 02	0.9736E 00	0.9501E 00
0.5800E 02	0.9688E 00	0.9477E 00
0.6000E 02	0.9643E 00	0.9455E 00
0.6200E 02	0.9599E 00	0.9434E 00
0.6400E 02	0.9557E 00	0.9413E 00
0.6600E 02	0.9518E 00	0.9394E 00
0.6800E 02	0.9481E 00	0.9375E 00
0.7000E 02	0.9447E 00	0.9359E 00
0.7200E 02	0.9415E 00	0.9343E 00
0.7400E 02	0.9387E 00	0.9329E 00
0.7600E 02	0.9361E 00	0.9316E 00
0.7800E 02	0.9339E 00	0.9305E 00
0.8000E 02	0.9320E 00	0.9296E 00
0.8200E 02	0.9305E 00	0.9288E 00
0.8400E 02	0.9292E 00	0.9282E 00
0.8600E 02	0.9284E 00	0.9277E 00
0.8800E 02	0.9278E 00	0.9275E 00
0.9000E 02	0.9277E 00	0.9274E 00

TM MAGNETIC CURRENTS FOR PHI = 0.0000E 00 0.0000E 00

POS.	REAL	IMAG	MAG
	APERTURE FACE 1 ...		
1	-0.3786073E 00	-0.1068542E 01	0.1133633E 01
2	-0.4921361E 00	-0.1327325E 01	0.1415623E 01
3	-0.4561581E 00	-0.1233211E 01	0.1314872E 01
4	-0.3424406E 00	-0.9786575E 00	0.1036839E 01
	APERTURE FACE 2 ...		
1	-0.1652662E 00	-0.4473922E 00	0.4769409E 00
2	-0.1963732E 00	-0.5275222E 00	0.5628873E 00
3	-0.1783029E 00	-0.4792310E 00	0.5113260E 00
	APERTURE FACE 3 ...		
1	-0.5145471E-01	-0.1149146E 00	0.1259085E 00
2	-0.6691802E-01	-0.1537555E 00	0.1676865E 00
3	-0.5205211E-01	-0.1163712E 00	0.1274821E 00

EXCITATIONS AND TRANSMISSION COEFFS.

0.0000E 00	0.0000E 00	0.1369E-02
0.0000E 00	0.4500E 02	0.5987E-03

TM GAIN AND FIELD PATTERNS

-0.9000E 02	0.1861E-12	0.3022E-06
-0.8800E 02	0.2301E-02	0.3360E-01
-0.8600E 02	0.9193E-02	0.6716E-01
-0.8400E 02	0.2065E-01	0.1007E 00
-0.8200E 02	0.3664E-01	0.1341E 00
-0.8000E 02	0.5708E-01	0.1674E 00
-0.7800E 02	0.8191E-01	0.2005E 00
-0.7600E 02	0.1110E 00	0.2334E 00
-0.7400E 02	0.1443E 00	0.2661E 00
-0.7200E 02	0.1817E 00	0.2986E 00
-0.7000E 02	0.2229E 00	0.3307E 00
-0.6800E 02	0.2679E 00	0.3626E 00
-0.6600E 02	0.3164E 00	0.3940E 00
-0.6400E 02	0.3683E 00	0.4251E 00
-0.6200E 02	0.4233E 00	0.4558E 00
-0.6000E 02	0.4813E 00	0.4859E 00
-0.5800E 02	0.5419E 00	0.5156E 00
-0.5600E 02	0.6049E 00	0.5448E 00
-0.5400E 02	0.6700E 00	0.5734E 00
-0.5200E 02	0.7369E 00	0.6013E 00
-0.5000E 02	0.8054E 00	0.6286E 00
-0.4800E 02	0.8750E 00	0.6552E 00
-0.4600E 02	0.9456E 00	0.6811E 00
-0.4400E 02	0.1017E 01	0.7063E 00
-0.4200E 02	0.1088E 01	0.7306E 00
-0.4000E 02	0.1159E 01	0.7541E 00
-0.3800E 02	0.1230E 01	0.7768E 00
-0.3600E 02	0.1299E 01	0.7985E 00
-0.3400E 02	0.1368E 01	0.8193E 00
-0.3200E 02	0.1435E 01	0.8391E 00
-0.3000E 02	0.1500E 01	0.8579E 00
-0.2800E 02	0.1563E 01	0.8756E 00
-0.2600E 02	0.1623E 01	0.8923E 00
-0.2400E 02	0.1680E 01	0.9078E 00
-0.2200E 02	0.1734E 01	0.9223E 00
-0.2000E 02	0.1784E 01	0.9355E 00
-0.1800E 02	0.1830E 01	0.9476E 00
-0.1600E 02	0.1872E 01	0.9585E 00
-0.1400E 02	0.1910E 01	0.9682E 00
-0.1200E 02	0.1944E 01	0.9766E 00
-0.1000E 02	0.1972E 01	0.9837E 00
-0.8000E 01	0.1996E 01	0.9895E 00
-0.6000E 01	0.2014E 01	0.9941E 00
-0.4000E 01	0.2027E 01	0.9974E 00
-0.2000E 01	0.2035E 01	0.9993E 00
0.0000E 00	0.2038E 01	0.1000E 01
0.2000E 01	0.2035E 01	0.9993E 00
0.4000E 01	0.2027E 01	0.9974E 00
0.6000E 01	0.2014E 01	0.9941E 00

0.8000E 01	0.1996E 01	0.9895E 00
0.1000E 02	0.1972E 01	0.9837E 00
0.1200E 02	0.1944E 01	0.9765E 00
0.1400E 02	0.1910E 01	0.9681E 00
0.1600E 02	0.1872E 01	0.9585E 00
0.1800E 02	0.1830E 01	0.9476E 00
0.2000E 02	0.1784E 01	0.9355E 00
0.2200E 02	0.1733E 01	0.9222E 00
0.2400E 02	0.1680E 01	0.9078E 00
0.2600E 02	0.1622E 01	0.8922E 00
0.2800E 02	0.1562E 01	0.8756E 00
0.3000E 02	0.1500E 01	0.8578E 00
0.3200E 02	0.1435E 01	0.8390E 00
0.3400E 02	0.1368E 01	0.8192E 00
0.3600E 02	0.1299E 01	0.7984E 00
0.3800E 02	0.1230E 01	0.7767E 00
0.4000E 02	0.1159E 01	0.7541E 00
0.4200E 02	0.1088E 01	0.7306E 00
0.4400E 02	0.1017E 01	0.7062E 00
0.4600E 02	0.9454E 00	0.6811E 00
0.4800E 02	0.8749E 00	0.6552E 00
0.5000E 02	0.8052E 00	0.6286E 00
0.5200E 02	0.7368E 00	0.6013E 00
0.5400E 02	0.6698E 00	0.5733E 00
0.5600E 02	0.6047E 00	0.5447E 00
0.5800E 02	0.5418E 00	0.5156E 00
0.6000E 02	0.4812E 00	0.4859E 00
0.6200E 02	0.4233E 00	0.4557E 00
0.6400E 02	0.3682E 00	0.4251E 00
0.6600E 02	0.3164E 00	0.3940E 00
0.6800E 02	0.2678E 00	0.3625E 00
0.7000E 02	0.2229E 00	0.3307E 00
0.7200E 02	0.1816E 00	0.2985E 00
0.7400E 02	0.1443E 00	0.2661E 00
0.7600E 02	0.1110E 00	0.2334E 00
0.7800E 02	0.8189E-01	0.2004E 00
0.8000E 02	0.5706E-01	0.1673E 00
0.8200E 02	0.3662E-01	0.1341E 00
0.8400E 02	0.2065E-01	0.1006E 00
0.8600E 02	0.9189E-02	0.6715E-01
0.8800E 02	0.2299E-02	0.3359E-01
0.9000E 02	0.1606E-09	0.8878E-05

3.1.3 Minimum Storage Requirements for Arrays

The minimum storage required for each array used in Program II is given here. Arrays which are execution-time dimensioned are not mentioned (i.e., array Y in subroutine YHSTM). Integers which are used to specify the array sizes are defined by:

NR = total number of unknown coefficients in Eq. (6-8).

NEX = number of excitations.

NU(i) = number of unknown magnetic current expansion functions existing on aperture face Γ_i.

NB = number of rectangular sub-regions by which region b is approximated.

$$NU_{max} = \max_{1 \le i \le NB+1} NU(i)$$

$$NT_{max} = \max_{1 \le i \le NB} NT(i)$$

NGQ = order of Gaussian quadrature formulas used for numerical integration.

NI = number of points in region c at which far field must be evaluated to obtain gain pattern.

The allocations required for the arrays in common blocks, which appear in several program segments, are given by:

GQI: DIMENSION A(NGQ), T(NGQ)

B: DIMENSION W(NB+1), H(NB), YL(NB), YR(NB), DC(NB+1), D(NB), N(NB+1), NT(NB)

The allocations required for arrays used in the subroutines are:

MAIN PGM:	COMPLEX Y(NR*(NR+1)/2), XI(NR*NEX),
	YHSC(NU(NB+1)), YHS(NU(NB+1)),
	XZ(NU(1)), YZ(NU(1))
	DIMENSION PHI(2*NEX), PIN(NEX), IJOB(6),
	NU(NB+1), PT(NEX)
MAT:	COMPLEX Y(NR*(NR+1)/2), YD1(NU_{max}*NU_{max}),
	YD2(NU_{max}*NU_{max}), XI(NR*NEX),
	YHSA(NU(1)), YHSC(NU(NB+1)), WK(NB+1),
	HK(NB), YLK(NB), YRK(NB), DCK(NB+1),
	DK(NB)
	DIMENSION NU(NB+1), MU(NB+1)
	PHI(2*NEX), PIN(NEX)
YB1 and YB2:	COMPLEX Y(NU_{max}*NU_{max}), ST(NT_{max})
EXTE2 and EXTM2:	COMPLEX XI(NR*NEX)
	DIMENSION PHI(2*NEX), PIN(NEX)
TRANS2:	COMPLEX YHS(NU(NB+1)), YAUX(NU(NB+1)),
	VC(NU(NB+1)), VM(NR*NEX)
	DIMENSION PHI(2*NEX), PIN(NEX),
	PT(NEX)
GAIN2:	COMPLEX XI(NR*NEX)
	DIMENSION PHI(2*NEX), FP(NI),
	FT(NI), GA(NI), PT(NEX)
GELS:	COMPLEX R(NR*NEX), AUX(NR*(NR+1)/2)

3.2 Function Subprograms HANK02 and HANK12

These function subprograms return the Hankel functions of the second kind, orders zero and one, for a real argument as

$$HANK02(x) = H_0^{(2)}(x)$$

$$HANK12(x) = H_1^{(2)}(x)$$

The polynomial approximations of Abramowitz and Stegun [2, 9.4.1 - 9.4.6] are used.

```
      COMPLEX FUNCTION HANK02(X)
      COMPLEX U/(0.,1.)/
10    FORMAT('-','WARNING--ARGUMENT OF HANK02 =',E17.7)
      IF(X.LE.0.0)WRITE(3,10)X
      BSJ0=1.0
      IF(X.EQ.0.) GO TO 2
      Z=ABS(X)
      IF(Z.GT.3.0)GO TO 1
      Y=Z*Z/9.0
      BSJ0=1.0+Y*(-2.2499997+Y*(1.2656208+Y*(-0.3163866+Y*(0.0444479+Y*(
     1-0.0039444+Y*0.00021)))))
      GO TO 2
1     W=3./Z
      F0=.79788456+W*(-.00000077+W*(-.0055274+W*(-.00009512+W*(.001
     137237+W*(-.00072805+W*.00014476)))))
      P0=.78539816+W*(.04166397+W*(.00003954+W*(-.00262573+W*(.0005
     14125+W*(.00029333-W*.00013558)))))
      BSJ0=F0*COS(Z-P0)/SQRT(Z)
2     CONTINUE
      BSY0=-1.0E75
      IF(X.EQ.0.) GO TO 3
      IF(Z.GT.3.) GO TO 4
      BSY0=0.63661977*ALOG(0.5*Z)*BSJ0    +0.36746691+Y*(0.60559366+Y*(-0
     1.7435C384+Y*(0.25300117+Y*(-0.04261214+Y*(0.00427916-Y*0.00024846)
     2))))
      GO TO 3
4     CONTINUE
      BSY0=F0*SIN(Z-P0)/SQRT(Z)
3     CONTINUE
      HANK02=BSJ0-U*BSY0
      RETURN
      END
```

```
      COMPLEX FUNCTION HANK12(X)
      COMPLEX U/(0.,1.)/
100   FORMAT('-','WARNING--ARGUMENT OF HANK12 =',E17.7)
      IF(X.LE.0.) WRITE(3,100) X
      BSJ1=C.0
      IF(X.EQ.0.) GC TO 2
      Z=ABS(X)
      IF(Z.GT.3.0) GC TO 1
        Y=Z*Z/9.0
        BSJ1=X*(0.5+Y*(-0.56249985+Y*(0.21093573+Y*(-0.03954289+Y*(0.00443
     1319+Y*(-0.00031761+Y*0.00001109))))))
      GC TO 2
1     W=3./Z
        F1=.79788456+W*(.00000156+W*(.01659667+W*(.00017105+W*(-.0024
     19511+W*(.00113653-W*.00020033)))))
        P1=.78539816+W*(-.12499612+W*(-.00005650+W*(.00637879+W*(-.00
     107434E+W*(-.00079824+W*.00029166)))))
        BSJ1=F1*SIN(Z-P1)/SQRT(Z)
        IF(X.LT.0.0)BSJ1=-BSJ1
  2     CONTINUE
        BSY1=-1.0E75
        IF(X.EQ.0.) GO TO 3
        IF(Z.GT.3.) GO TO 4
        BSY1=(-0.63661977+Y*(0.2212091+Y*(2.1682709+Y*(-1.3164827+Y*(0.312
     13951+Y*(-C.0400976+Y*0.0027873))))))/Z+0.63661977*ALOG(0.5*Z)*BSJ1
      GO TO 3
4     BSY1=-F1*COS(Z-P1)/SQRT(Z)
3     CONTINUE
    HANK12=BSJ1-U*BSY1
      RETURN
      END
```

3.3 Subroutines EXTE2 and EXTM2

The subroutines compute the excitation vector for the right hand side of Eq. (6-8) for the TE case (EXTE2) or the TM case (EXTM2). This is done for each excitation source desired for a given slit problem and the result is stored in array XI. The argument parameters are defined by

Input: N = number of magnetic current expansion functions on aperture face Γ_1.

NEX = number of excitations.

PHI = excitation array defined in Section 2.1.1.

DCK = k_a*DC(1) = k_a W_1/N_1

$WA = k_a*W(1)$

$ETA = \eta_a/\eta_o$

$KA = k_a/k_o$

NR = total number of unknown coefficients in Eq. (6-8).

$WAVNO = k_o$

Output: XI = array containing all of the excitation vectors (right hand sides) of Eq. (6-8).

PIN(j) = $\eta_o k_o$ times the time average power incident upon aperture face Γ_1 when the jth source is normally incident.

The possible excitations here consist of a plane wave incident at an angle ϕ^i measured with respect to the negative x axis and a line source placed at coordinates (x_s, y_s) in region a. The coordinates of Fig. 1 are used where $x_s < 0$ and $-90° \leq \phi^i \leq 90°$. These subroutines are simply specializations of TMEXC and TEEXC in Section 2.4 where Eq. (6-15) is computed for the TE case (EXTE2 using Eq. (6-28)) and for the TM case (EXTM2 using Eq. (6-40)). The normal incident time average power, PIN, is computed for each excitation and is the same as in Section 2.4.

```
SUBROUTINE EXTM2(XI,N,NEX,PHI,DCK,WA,ETA,KA,NR,PIN,WAVNO)
COMPLEX XI(5000),U,CEXP,HANK02,HANK12,FE,SUM1,SUM2
COMMON /GQI/A(10),T(10),NGQ
DIMENSION PHI(100),PIN(100)
REAL KA
DATA PI/3.141593/,U/(0.,1.)/
D2=DCK/2.
WA2=WA/2.
K=1
KC=1
NP1=N+1
```

```
      DO 1 J=1,NEX
      IF(PHI(K).NE.0.) GO TO 10
      IF(PHI(K+1).EQ.0.) GO TO 4
      CPHI=COS(PHI(K+1)*PI/180.)
      SPHI=SIN(PHI(K+1)*PI/180.)
      FE=-DCK*CPHI*(SIN(SPHI*D2)/(SPHI*C2))**2/(ETA*KA)
      DO 2 I=1,N
      XI(KC)=2.*FE*CEXP(U*(I*DCK-WA2)*SPHI)
      KC=KC+1
2     CONTINUE
      GO TO 5
4     DO 3 I=1,N
      XI(KC)=-2.*DCK/(ETA*KA)
      KC=KC+1
3     CONTINUE
 5    PIN(J)=WA/(ETA*KA)
      GO TO 11
10    CONTINUE
      XS=WAVNO*KA*PHI(K)
      YS=WAVNO*KA*PHI(K+1)
      XS2=XS*XS
      RS=SQRT(XS2+(WA2-YS)**2)
      FE=HANKO2(RS)
      DO 7 I=1,N
      SUM1=0.
      SUM2=0.
      DO 8 L=1,NGQ
      ARG1=SQRT(XS2+(DCK*(T(L)+I-.5)-YS)**2)
      ARG1=SQRT(XS2+(DCK*(T(L)+I+.5)-YS)**2)
      SUM1=SUM1+A(L)*(T(L)+.5)*HANK12(ARG1)/ARG1
      SUM2=SUM2+A(L)*(.5-T(L))*HANK12(ARG2)/ARG2
8     CONTINUE
      XI(KC)=2.*XS*DCK*(SUM1+SUM2)/(ETA*KA*U*FE)
      KC=KC+1
7     CONTINUE
      THET=2.*ATAN(WA2/RS)
      PIN(J)=2.*THET/(PI*ETA*KA*CABS(FE)**2)
11    DO 6 I=NP1,NR
      XI(KC)=0.
      KC=KC+1
6     CONTINUE
      K=K+2
1     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EXTE2(XI,N,NEX,PHI,DCK,WA,ETA,KA,NR,PIN,WAVNO)
      COMPLEX XI(5000),U,HANK02,HANK12,FE,SUM,CEXP
      COMMON /GCI/A(10),T(10),NGQ
      DIMENSION PHI(100),PIN(100)
      REAL KA
      DATA PI/3.141593/,U/(0.,1.)/
      D2=DCK/2.
      WA2=WA/2.
      K=1
      KC=1
      NP1=N+1
      DO 1 J=1,NEX
      IF(PHI(K).NE.0.) GO TO 10
      IF(PHI(K+1).EQ.0.) GO TO 4
      SPHI=SIN(PHI(K+1)*PI/180.)
      FE=DCK*SIN(SPHI*D2)/(SPHI*D2*ETA*KA)
      DO 2 I=1,N
      XI(KC)=2.*FE*CEXP(U*((I-.5)*DCK-WA2)*SPHI)
      KC=KC+1
2     CONTINUE
      GO TO 5
4     DO 3 I=1,N
      XI(KC)=2.*DCK/(ETA*KA)
      KC=KC+1
3     CONTINUE
5     PIN(J)=WA/(ETA*KA)
      GO TO 11
10    CONTINUE
      XS=WAVNO*KA*PHI(K)
      YS=WAVNO*KA*PHI(K+1)
      XS2=XS*XS
      RS=SQRT(XS2+(WA2-YS)**2)
      FE=HANK12(RS)
      DO 7 I=1,N
      SUM=0.
      DO 8 L=1,NGQ
      ARG=SQRT(XS2+((T(L)+I-.5)*DCK-YS)**2)
      SUM=SUM+A(L)*HANK02(ARG)
8     CONTINUE
      XI(KC)=2.*U*DCK*SUM/(ETA*KA*FE)
      KC=KC+1
7     CONTINUE
      THET=2.*ATAN(WA2/RS)
      PIN(J)=2.*THET/(PI*ETA*KA*CABS(FE)**2)
11    DO 6 I=NP1,NR
      XI(KC)=0.
      KC=KC+1
6     CONTINUE
      K=K+2
1     CONTINUE
      RETURN
      END
```

3.4 Function Subprograms I4MN and ALPHA

ALPHA is the same as that described in Section 2.6 except that the argument is now real. I4MN is used to compute the half space admittance matrix for the TM case and is a specialization of Eq. (6-31). The argument parameters are defined by

DCK = γ = electrical length of subsection Δ_q.

ICM = subscript m.

ICN = subscript n

ISM = integer p

ISN = integer q

```
      COMPLEX FUNCTION I4MN(DCK,ICM,ICN,ISM,ISN)
      COMMCN /GQI/A(10),T(10),NGC
      COMPLEX HANK02,HANK12,SUM,ALPHA,AX,AY
      CD=ISM*ISN/DCK/DCK
      SUM=0.
      IF(ICM.EC.ICN) GC TO 2
      RMN=(ICM-ICN)*DCK
      CO 1 I=1,NGQ
      CO 1 J=1,NGQ
      ARG=AES(CCK*T(I)-CCK*T(J)+RMN)
      F=(ISN*T(I)+.5)*(ISN*T(J)+.5)-DC
      SUM=SUM+A(I)*A(J)*F*HANK02(ARG)
1     CONTINUE
      I4MN=SUM*CCK*DCK/2.
      RETURN
2     CO 3 I=1,NGQ
      F1=T(I)+.5
      F2=.5-T(I)
      AX=((ISM*T(I)+.5)*(ISN*T(I)+.5)-DC)*(ALPHA(F1*DCK)+ALPHA(F2*DCK))
      AY=ISN*(ISM*T(I)+.5)*(F2*HANK12(F2*DCK)-F1*HANK12(F1*DCK))
      SUM=SUM+A(I)*(AX+AY)
3     CONTINUE
      I4MN= DCK*SUM/2.
      RETURN
      END
```

```
COMPLEX FUNCTION ALPHA(Z)
COMPLEX HANK02,HANK12
Z2=Z*Z
H0=Z*(1.-Z2/9.*(1.-Z2/25.*(1.-Z2/49.)))
H1=(Z2/3.)*(1.-Z2/15.*(1.-Z2/35.*(1.-Z2/63.)))
ALPHA=Z*HANK02(Z)*(1.-H1)+Z*HANK12(Z)*H0
RETURN
END
```

3.5 Subroutines YHSTE and YHSTM

These subroutines compute the first column of the symmetric Toeplitz half space admittance matrices whose elements are given by Eq. (6-17) for the TE case and Eq. (6-30) for the TM case. The argument parameters are defined by

Input: N = number of expansion functions on aperture face adjacent to a half space region.

DCK = subsection length of aperture face, Δ_q, multiplied by wavenumber of half space.

WAV = wavenumber of half space divided by wavenumber of free space.

ET = impedance of half space divided by impedance of free space.

Output: Y = first column of symmetric Toeplitz admittance matrix.

```
      SUBROUTINE YHSTE(Y,N,DCK,WAV,ET)
      COMMON /GQI/A(10),T(10),NGQ
      COMPLEX ALPHA,HANK02,Y(N),SUM
      CONST=1./(2.*WAV*ET)
      SUM=0.
      H=DCK/2.
      DO 1 I=1,NGQ
      SUM=SUM+A(I)*(ALPHA(H-T(I)*DCK)+ALPHA(H+T(I)*DCK))
    1 CONTINUE
      Y(1)=DCK*SUM*CONST
      DO 3 IR=2,N
      SUM=0.
      DO 2 I=1,NGQ
      DO 2 J=1,NGQ
      SUM=SUM+A(I)*A(J)*HANK02(ABS(DCK*(T(I)-T(J)+1-IR)))
    2 CONTINUE
      Y(IR)=DCK*DCK*SUM*CONST
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE YHSTM(Y,N,DCK,WAV,ET)
      COMPLEX Y(N),I4MN
      DO 1 I=1,N
      Y(I)=I4MN(DCK,1,I,1,1)+I4MN(DCK,1,I+1,1,-1)
     1+I4MN(DCK,2,I,-1,1)+I4MN(DCK,2,I+1,-1,-1)
      Y(I)=Y(I)/(ET*WAV)
    1 CONTINUE
      RETURN
      END
```

3.6 Subroutine MAT

This subroutine forms the upper right triangle of the matrix of coefficients in Eq. (6-8) and the vectors of the right hand side by calling EXTE2 or EXTM2. The argument parameters are defined by

Input: ID = integer option variable, ID = 1 for TE case and ID = 2 for TM case.

NEX = number of excitations.

PHI = excitation array defined in Section 2.1.1.

Output: Y = upper right triangle of coefficient matrix in Eq. (6-8), stored columnwise.

NR = total number of unknowns or rank of matrix in Eq. (6-8).

XI = excitation vectors computed in EXTE2 or EXTM2.

YHSC = first column of matrix $[Y^c]$.

NU = integer array, the qth element of which equals the number of magnetic current expansion functions on Γ_q.

PIN(j) = $k_o \eta_o P_{iN}$ computed in EXTE2 or EXTM2 for the jth excitation.

The number of unknowns for vectors $\vec{V}_1$, $\vec{V}_2$,... etc. are computed in DO loop 2 and stored in array NU. NR is the number of rows in the complete system of equations to be solved. MU is an integer array used as an index along the diagonal of the coefficient matrix. It is computed by

$$MU(q) = \frac{1}{2}\left(\sum_{i=1}^{q} N_i\right)\left(\sum_{i=1}^{q} N_i + 1\right)$$

for q = 1,2,..., NB+1. The upper right triangular portion of matrices $[Y_{11}^{b1}]$ and $[Y_{22}^{bQ}]$ are formed by calling YB1 and they are stored in YD1 and YD2 respectively. DO loop 5 forms the matrix $[Y^a + Y_{11}^{b1}]$ and stores the result in Y. DO loop 14 forms the matrix $[Y_{22}^{bQ} + Y^c]$ and stores the result in the appropriate locations in Y. DO loop 7 computes the matrices $[Y_{12}^{b1}]$, $[Y_{12}^{b2}]$,..., $[Y_{12}^{bQ}]$ by calling YB2 and stores them successively in Y. DO loop 10 forms the matrices

$$[Y_{22}^{b1} + Y_{11}^{b2}],\ [Y_{22}^{b2} + Y_{11}^{b3}],\ \ldots,\ [Y_{22}^{bQ-1} + Y_{11}^{bQ}]$$

and stores them in array Y.

```
      SUBROUTINE MAT(Y,NR,ID,XI,YHSC,NEX,PHI,NU,PIN)
      COMPLEX Y(4000),YD1(400),YD2(400),XI(5000)
      COMPLEX YHSA(40),YHSC(40),ETANB,WAVNB
      COMMON /B/W(10),H(10),YL(10),YR(10),DC(10),D(10),N(10),NT(10),NB
      COMMON /E/ETANA,ETANB,ETANC,WAVNA,WAVNB,WAVNC,WAVNO
      COMPLEX WK(10),HK(10),YLK(10),YRK(10),DCK(10),DK(10)
      INTEGER NU(10),MU(10)
      DIMENSION PHI(100),PIN(100)
      NB1=NB+1
      DO 2 I=1,NB1
      NU(I)=N(I)-ID+1
2     CONTINUE
      NR=0
      NK=0
      DO 3 I=1,NB1
      NK=NK+NU(I)
      MU(I)=NK*(NK+1)/2
      NR=NR+NU(I)
3     CONTINUE
      NN=MU(NB1)
      DO 1 I=1,NN
      Y(I)=0.
1     CONTINUE
      WKA=W(1)*WAVNA
      WKC=W(NB1)*WAVNC
      DCA=DC(1)*WAVNA
      DCC=DC(NB1)*WAVNC
      IF(ID.NE.1) GO TO 12
```

```
      CALL  YHSTE(YHSA,NU(1),DCA,WAVNA,ETANA)
      CALL  YHSTE(YHSC,NU(NB1),DCC,WAVNC,ETANC)
      CALL  EXTE2(XI,NU(1),NEX,PHI,DCA,WKA,ETANA,WAVNA,NR,PIN,WAVNO)
      GO TO 13
12    CALL  YHSTM(YHSA,NU(1),DCA,WAVNA,ETANA)
      CALL  YHSTM(YHSC,NU(NB1),DCC,WAVNC,ETANC)
      CALL  EXTM2(XI,NU(1),NEX,PHI,DCA,WKA,ETANA,WAVNA,NR,PIN,WAVNO)
13    CONTINUE
      DO 4  I=1,NB
      WK(I)=W(I)*WAVNB
      HK(I)=H(I)*WAVNB
      YLK(I)=YL(I)*WAVNB
      YRK(I)=YR(I)*WAVNB
      DCK(I)=DC(I)*WAVNB
      DK(I)=D(I)*WAVNB
4     CONTINUE
      WK(NB1)=W(NB1)*WAVNB
      DCK(NB1)=WK(NB1)/N(NB1)
      CALL  YB1(YD1,N(1),NT(1),ID,DK(1),DCK(1),HK(1),YLK(1),
     1ETANB,WAVNB)
      CALL  YB1(YD2,N(NB1),NT(NB),ID,DK(NB),DCK(NB1),HK(NB),
     1YRK(NB),ETANB,WAVNB)
      K=1
      N1=NU(1)
      DO 5  J=1,N1
      DO 5  I=1,J
      Y(K)=YD1(K)+YHSA(J-I+1)
      K=K+1
5     CONTINUE
      M1=MU(NB)
      NK=NR-NU(NB1)
      K=1
      L=NK
      N1=NU(NB1)
      DO 14  J=1,N1
      DO 6  I=1,J
      Y(K+M1+L)=YD2(K)+YHSC(J-I+1)
      K=K+1
6     CONTINUE
      L=L+NK
14    CONTINUE
      NK=0
      DO 7  I=1,NB
      CALL  YB2(YD1,NT(I),N(I+1),N(I),ID,DK(I),DCK(I),DCK(I+1),
     1HK(I),YLK(I),YRK(I),ETANB,WAVNB)
      L=1
      K=MU(I)+1
      NC=NU(I+1)
      NIR=NU(I)
      DO 8  J=1,NC
      K=K+J-1+NK
      DO 9  IM=1,NIR
```

```
      Y(K)=YD1(L)
      L=L+1
      K=K+1
8     CONTINUE
      NK=NK+NU(I)
7     CONTINUE
      IF(NB.EQ.1) RETURN
      NK=NU(1)
      DO 10 I=2,NB
      CALL YB1(YD1,N(I),NT(I-1),ID,DK(I-1),DCK(I),HK(I-1),
     1YRK(I-1),ETANB,WAVNB)
      CALL YB1(YD2,N(I),NT(I),ID,DK(I),CCK(I),HK(I),
     1YLK(I),ETANB,WAVNB)
      M1=MU(I-1)
      K=1
      L=NK
      N1=NU(I)
      DO 11 J=1,N1
      DO 15 IM=1,J
      Y(K+M1+L)=YD1(K)+YD2(K)
      K=K+1
15    CONTINUE
      L=L+NK
11    CONTINUE
      NK=NK+NU(I)
10    CONTINUE
      RETURN
      END
```

3.7 Subroutine GELS

This subroutine solves a system of simultaneous linear equations with a symmetric coefficient matrix by Gaussian elimination. A detailed explanation of the algorithm appears in [5]. The argument parameters are:

R = M × N right hand side matrix containing excitation vectors.

A = upper right triangular part of the M × M coefficient matrix which is destroyed during execution.

M = the number of equations in the system

N = the number of right hand side excitation vectors.

M2 = M*(M+1)/2

An error message may also be printed out to indicate round off error or ill-conditioning for some applications but it is unlikely to occur here. This is explained in detail in [5].

```
      SUBROUTINE GELS(R,A,M,N,M2)
      COMPLEX A(M2)
      COMPLEX R(5000),AUX(4000)
      COMPLEX PIVI,TB
100   FORMAT('1','WARNING---ERROR CODE IER = ',I5)
      EPS=1.E-07
      IF(M)24,24,1
    1 IER=0
      PIV=0.
      L=0
      DO 3 K=1,M
      L=L+K
      TBA=CABS(A(L))
      IF(TBA-PIV) 3,3,2
  2   PIV=TBA
      I=L
      J=K
    3 CONTINUE
      TOL=EPS*PIV
      LST=0
      NM=N*M
      LEND=M-1
      DO 18 K=1,M
      IF(PIV)24,24,4
    4 IF(IER)7,5,7
    5 IF(PIV-TOL)6,6,7
    6 IER=K-1
    7 LT=J-K
      LST=LST+K
      PIVI=1./A(I)
      DO 8 L=K,NM,M
      LL=L+LT
      TB=PIVI*R(LL)
      R(LL)=R(L)
8     R(L)=TB
      IF(K-M)9,19,19
    9 LR=LST+(LT*(K+J-1))/2
      LL=LR
      L=LST
      DO 14 II=K,LEND
      L=L+II
      LL=LL+1
      IF(L-LR)12,10,11
   10 A(LL)=A(LST)
      TB=A(L)
      GO TO 13
```

```
11 LL=L+LT
12 TB=A(LL)
   A(LL)=A(L)
13 AUX(II)=TE
14 A(L)=PIVI*TB
   A(LST)=LT
   PIV=0.
   LLST=LST
   LT=0
   DO 18 II=K,LEND
   PIVI=-AUX(II)
   LL=LLST
   LT=LT+1
   DO 15 LLD=II,LEND
   LL=LL+LLD
   L=LL+LT
15 A(L)=A(L)+PIVI*A(LL)
   LLST=LLST+II
   LR=LLST+LT
   TBA=CABS(A(LR))
   IF(TBA-PIV) 17,17,16
16 PIV=TEA
   I=LR
   J=II+1
17 DO 18 LR=K,NM,M
   LL=LR+LT
18 R(LL)=R(LL)+PIVI*R(LR)
19 IF(LEND) 24,23,20
20 II=M
   DO 22 I=2,M
   LST=LST-II
   II=II-1
   L=A(LST)+.5
   DO 22 J=II,NM,M
   TB=R(J)
   LL=J
   K=LST
   DO 21 LT=II,LEND
   LL=LL+1
   K=K+LT
21 TB=TB-A(K)*R(LL)
   K=J+L
   R(J)=R(K)
22 R(K)=TB
23 RETURN
24 IER=-1
   WRITE(3,100) IER
   RETURN
   END
```

3.8 Subroutines YB1 and YB2

Subroutine YB1 computes the matrices $[Y_{11}^{bq}]$ and $[Y_{22}^{bq}]$ for the TE case (Eqs. (6-24) and (6-25)) or the TM case (Eqs. (6-36) and (6-37)). Subroutine YB2 computes the matrix $[Y_{12}^{bq}]$ for the TE case (Eq. (6-26)) or the TM case (Eq. (6-38)). These two subroutines are quite similar and argument parameters are defined in terms of the variables appearing in Chapter 6 of [1].

Input: NT = number of terms at which infinite summations are truncated.

N = number of expansion functions on Γ_q or Γ_{q+1}.

M = number of expansion functions on Γ_{q+1}.

ID = integer option variable, ID = 1 for TE case and ID = 2 for TM case.

DK = $k_b d_q$

DCK = $k_b(\Delta_q$ or $\Delta_{q+1})$

DCKM = $k_b \Delta_q$

DCKN = $k_b \Delta_{q+1}$

HK = $k_b h_q$

YK = $k_b(y_{\ell q}$ or $y_{\ell q+1})$

YKM = $k_b y_{\ell q}$

YKN = $k_b y_{rq}$

ETB = η_b / η_o

KB = k_b / k_o

Output: Y = appropriate submatrix of Eq. (6-8).

```
      SUBROUTINE YB1(Y,N,NT,ID,DK,DCK,HK,YK,ETB,KB)
      COMPLEX Y(400),ST(200),SIN2
      COMPLEX DK,HK,YK,ETB,KB,YB,KBD,CXP,SPS
      COMPLEX CSIN,CCOS,CSQRT,U,CONST,YS,DCK,CS,SN
      DATA U/(0.,1.)/,PI/3.141593/
      CONST=-U*DCK*DCK/(ETB*KB*HK)
      IF(ID.NE.1) GO TO 3
      IF(ABS(AIMAG(DK)).GE.50.) GO TO 1
      YB=CONST*CCOS(DK)/CSIN(DK)
      GO TO 2
1     MF=1
      IF(AIMAG(DK).GT.0.) MF=-1
      YB=MF*CONST*U
2     CONTINUE
3     DO 5 IP=1,NT
      CXP=CSQRT(1.-(IP*PI/HK)**2)
      KBD=CXP*DK
      SPS=(IP*PI*DCK/2./HK)
      SIN2=(CSIN(SPS)/SPS)**2
      IF(ABS(AIMAG(KBD)).GE.50.) GO TO 6
      IF(ID.NE.1) GO TO 7
      ST(IP)=SIN2*CCOS(KBD)/(CSIN(KBD)*CXP)
      GO TO 10
7     ST(IP)=CXP*SIN2*SIN2*CCOS(KBD)/CSIN(KBD)
      GO TO 10
6     MF=1
      IF(AIMAG(KBD).GT.0.) MF=-1
      IF(ID.NE.1) GO TO 8
      ST(IP)=MF*U*SIN2/CXP
      GO TO 10
8     ST(IP)=MF*U*CXP*SIN2*SIN2
10    CONTINUE
5     CONTINUE
      CONST=CONST*2.
      NU=N-ID+1
      K=1
      DO 15 I=1,NU
      DO 15 J=1,I
      YS=0.
      IF(ID.NE.1) GO TO 17
      DO 16 IP=1,NT
      CS=CCOS(IP*PI*(YK+(I-.5)*DCK)/HK)*
     1CCOS(IP*PI*(YK+(J-.5)*DCK)/HK)
      YS=YS+ST(IP)*CS
16    CONTINUE
      Y(K)=YB+YS*CONST
      K=K+1
      GO TO 18
17    DO 19 IP=1,NT
      SN=CSIN(IP*PI*(YK+I*DCK)/HK)*CSIN(IP*PI*(YK+J*DCK)/HK)
      YS=YS+ST(IP)*SN
19    CONTINUE
      Y(K)=YS*CONST
      K=K+1
18    CONTINUE
15    CONTINUE
      RETURN
      END
```

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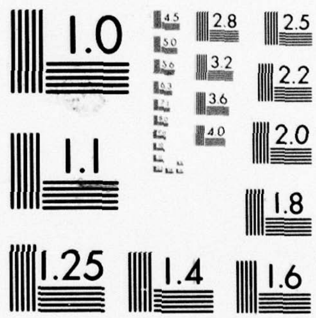

MICROCOPY RESOLUTION TEST CHART
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```
      SUBROUTINE YB2(Y,NT,N,M,ID,DK,DCKM,DCKN,HK,YKM,YKN,ETB,KB)
      COMPLEX Y(400),ST(200),SPM,SPN,CS,SN,SIN2
      COMPLEX DK,HK,YKM,YKN,ETB,KB,YB,KBD,CXP
      COMPLEX CSIN,CCOS,CSQRT,U,CONST,YS,DCKM,DCKN
      DATA U/(0.,1.)/,PI/3.141593/
      CONST= U*DCKM*DCKN/(ETB*KB*HK)
      IF(ID.NE.1) GO TO 2
      IF(ABS(AIMAG(DK)).GE.50.) GO TO 1
      YB=CONST/CSIN(DK)
      GO TO 2
1     YB=0.
2     DO 5 IP=1,NT
      SPM=IP*PI*DCKM/2./HK
      SPN=IP*PI*DCKN/2./HK
      SIN2=CSIN(SPM)*CSIN(SPN)/SPM/SPN
      CXP=CSQRT(1.-(IP*PI/HK)**2)
      KBD=CXP*DK
      IF(ABS(AIMAG(KBD)).GE.50.) GO TO 6
      IF(ID.NE.1) GO TO 7
      ST(IP)=SIN2/(CSIN(KBD)*CXP)
      GO TO 10
7     ST(IP)=CXP*SIN2*SIN2/CSIN(KBD)
      GO TO 10
6     ST(IP)=0.
10    CONTINUE
5     CONTINUE
      CONST=CONST*2.
      NU=N-ID+1
      MU=M-ID+1
      K=1
      DO 15 J=1,NU
      DO 15 I=1,MU
      YS=0.
      IF(ID.NE.1) GO TO 17
      DO 16 IP=1,NT
      CS=CCOS(IP*PI*(YKM+(I-.5)*DCKM)/HK)*
     1CCOS(IP*PI*(YKN+(J-.5)*DCKN)/HK)
      YS=YS+ST(IP)*CS
16    CONTINUE
      Y(K)=YB+YS*CONST
      K=K+1
      GO TO 18
17    DO 19 IP=1,NT
      SN=CSIN(IP*PI*(YKM+I*DCKM)/HK)*CSIN(IP*PI*(YKN+J*DCKN)/HK)
      YS=YS+ST(IP)*SN
19    CONTINUE
      Y(K)=YS*CONST
      K=K+1
18    CONTINUE
15    CONTINUE
      RETURN
      END
```

3.9 Subroutines TRANS2 and GAIN2

These subroutines are simply specializations of TRANS1 and GAIN1 which are explained in Sections 2.9 and 2.10. They are used to compute the transmission coefficient and the gain and normalized field patterns of region c.

```
      SUBROUTINE TRANS2(YHS,PIN,PHI,NEX,VM,ID,PT)
      COMMON /B/W(10),H(10),YL(10),YR(10),DC(10),D(10),N(10),NT(10),NB
      COMPLEX YHS(40),YAUX(40),VC(40),VM(5000)
      COMPLEX CONJG,SUM,S
      DIMENSION PHI(100),PIN(100),PT(100)
      DATA PI/3.141593/
100   FORMAT('-','EXCITATIONS AND TRANSMISSION COEFFS.')
101   FORMAT(' ',3E11.4)
      NSK=0
      DO 2 I=1,NB
      NSK=NSK+N(I)+1-ID
2     CONTINUE
      NC=N(NB+1)+1-ID
      NR=NSK+NC
      DO 1 I=1,NC
      YAUX(I)=CONJG(YHS(I))
1     CONTINUE
      WRITE(3,100)
      K=NSK
      DO 10 J=1,NEX
      DO 3 I=1,NC
      VC(I)=CONJG(VM(K+I))
3     CONTINUE
      S=0.
      DO 4 I=1,NC
      S=S+VM(K+I)*VC(I)
4     CONTINUE
      SUM=YAUX(1)*S
      DO 5 L=2,NC
      S=0.
      NLI=NC-L+1
      DO 6 I=1,NLI
      S=S+VM(K+I)*VC(L+I-1)+VM(K+L+I-1)*VC(I)
6     CONTINUE
      SUM=SUM+YAUX(L)*S
5     CONTINUE
      PT(J)=REAL(SUM)
      T=PT(J)/PIN(J)
      WRITE(3,101) PHI(2*J-1),PHI(2*J),T
      WRITE(2,101) PHI(2*J-1),PHI(2*J),T
      K=K+NR
10    CONTINUE
      RETURN
      END
```

```
      SUBRCUTINE GAIN2(XI,PHI,DPHI,NI,PT,ID,NEX)
      COMMCN /B/W(10),H(10),YL(10),YR(10),DC(10),D(10),N(10),NT(10),NB
      COMMCN /E/ETANA,ETANB,ETANC,WAVNA,WAVNB,WAVNC,WAVNO
      COMPLEX XI(5000),U,CEXP,S,YM,ETANE,WAVNB
      DIMENSION PHI(100),FP(100),FT(100),GA(100),PT(100)
      CATA PHO/-1.570796/,U/(0.,1.)/,ETA0/377./
100   FORMAT('1','TE GAIN AND FIELC PATTERNS')
101   FORMAT('1','TM GAIN AND FIELD PATTERNS')
102   FORMAT(' ',3E15.4)
      NB1=NE+1
      NSK=0
      CO 1 I=1,NB
      NSK=NSK+N(I)+1-IC
1     CONTINUE
      NC=N(NB1)+1-ID
      NR=NSK+NC
      WK2=W(NB1)*WAVNC/2.
      CPH=-CPHI*PHC/90.
      PH=PHC
      D2=DC(NB1)*WAVNC/2.
      CPH=-1.
      CO 2 I=1,NI
      SN=SIN(PH)
      IF(IC.EQ.2) CPH=-COS(PH)
      IF(PH.EQ.0.) GC TC 3
      FT(I)=-SIN(D2*SN)/(D2*SN)
      IF(ID.EQ.2) FT(I)=CPH*FT(I)**2
      GO TC 4
3     FT(I)=CPH
4     CONTINUE
      PH=PH+DPH
2     CONTINUE
      CONST=ETA0*2.*D2**2/WAVNC
      K=NSK
      CO 10 J=1,NEX
      PH=PHC
      CO 6 I=1,NI
      SN=SIN(PH)
      S=0.
      CO 7 L=1,NC
      YM=((2*L+ID-2)*D2-WK2)*SN*U
      S=S+XI(K+L)*CEXP(YM)*FT(I)
7     CONTINUE
      F=2.*D2*CABS(S)
      FP(I)=F
      GA(I)=F*F/(2.*WAVNC*ETANC*PT(J))
      PH=PH+DPH
6     CONTINUE
      FM=FP(1)
      CO 8 I=2,NI
      IF(FM.LT.FP(I)) FM=FP(I)
8     CONTINUE
```

```
      DO 9 I=1,NI
      FP(I)=FP(I)/FM
9     CONTINUE
      PH=-90.
      IF(ID.EQ.1) WRITE(3,100)
      IF(ID.EQ.2) WRITE(3,101)
      DO 5 I=1,NI
      WRITE(3,102) PH,GA(I),FP(I)
      WRITE(2,102) PH,GA(I),FP(I)
      PH=PH+DPHI
5     CONTINUE
      K=K+NR
10    CONTINUE
      RETURN
      END
```

Chapter 4

PROGRAM III - MODAL SOLUTION FOR RECTANGULAR SLIT, SLIT IMPEDANCE, AND PLANE WAVE APPROXIMATIONS

The purpose of this computer program is to investigate several approximate methods for treating a filled slit in a conducting plane of finite thickness when the slit cross section has a simple rectangular shape as shown in Fig. 5. The specialization of the modal program in Chapter 3 is made so that an accurate method of solution is always obtained. The cases of interest are when the slit width w is small compared to a wavelength in region c. It is shown in Section 5.4 of [1] that the transmitted fields may be written in terms of a slit impedance for the TE case and a slit polarizability for the TM case. Also when the material filling the slit is dense, the plane wave approximation developed in Chapter 7 of [1] is investigated.

Some subroutines which are required in the main program are exactly the same as those used in Program II. These are listed as follows:

EXTE2

EXTM2

HANK02

HANK12

I4MN

ALPHA

YHSTE

YHSTM

GELS

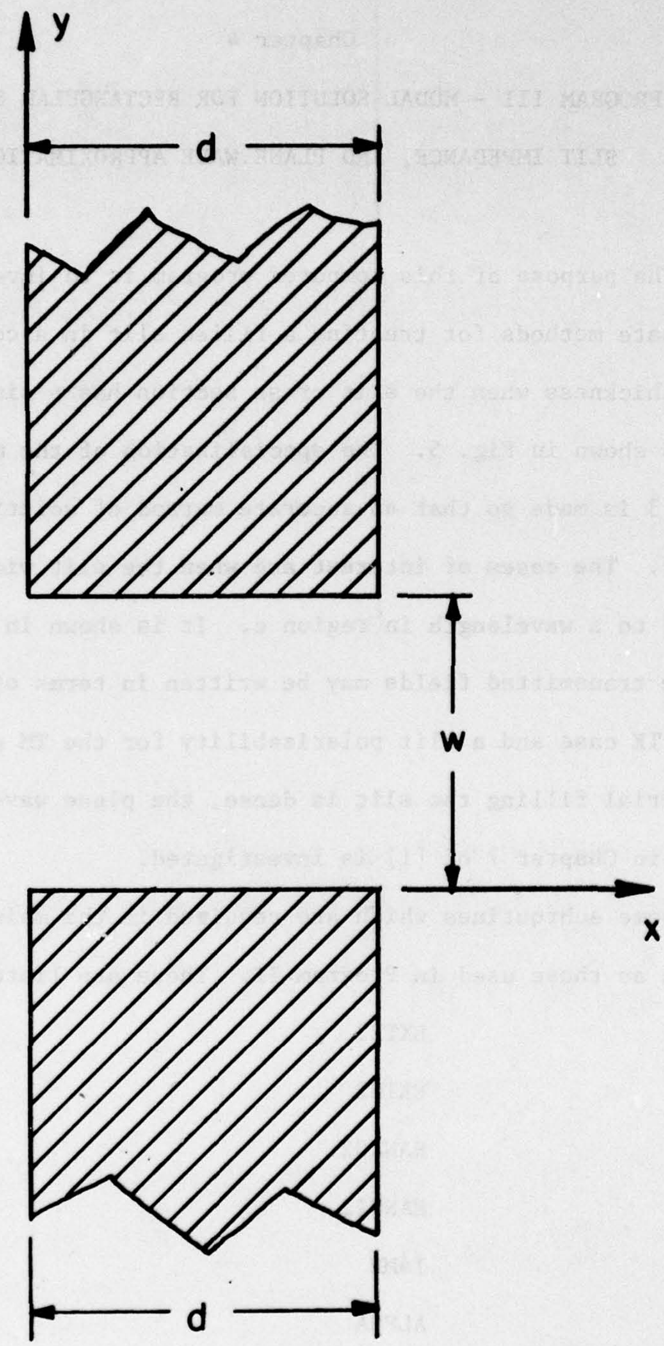

Fig. 5. Slit of rectangular cross section for Program III.

and will not be described in this chapter. Subroutines GAIN2 and TRANS2, as they are used in Program III, are slightly modified and are listed, though they are very similar to the ones used in Program II. The rest of the subroutines are described and listed in the successive sections.

Table 6. Arrangement of data cards for Program III, N1 = NEX + NGQ + 1

Data Card Number	Format Number	Information to be Typed on Card
1	101	NGQ
2	102	A(1), T(1)
.	.	.
.	.	.
.	.	.
1 + NGQ	102	A(NGQ), T(NGQ)
2 + NGQ	101	IJOB(1), IJOB(2),..., IJOB(8)
3 + NGQ	100	PHI(1), PHI(2)
.	.	. .
.	.	. .
.	.	. .
1 + N1	100	PHI(2*NEX-1), PHI(2*NEX)
2 + N1	100	NMUA, NEPSA
3 + N1	100	NMUB, NEPSB
4 + N1	100	NMUC, NEPSC
5 + N1	100	W, FMC, DPHI
6 + N1	101	N, NT, ND, NI
7 + N1	100	D(1)
.	.	.
.	.	.
.	.	.
6 + N1 + ND	100	D(ND)

4.1.1 Required Input Data and Main Program Description

The input data required by the user is read from a sequence of data cards in the main program according to the format statements:

```
100  FORMAT(6E11.4)
101  FORMAT(8I5)
102  FORMAT(2E20.7)
```

The sequence of data cards is arranged according to Table 6. The variables NGQ, A, T, PHI, NMUA, NEPSA, NMUB, NEPSB, NMUC, NEPSC, FMC, DPHI, and NI have exactly the same meaning as they do in Section 3.1.1 for Program II. The rest of the variables are defined by:

W = slit width

N = number of subsections of equal length into which both aperture faces are divided.

NT = Number of terms at which infinite sums, used for computing admittance matrices, are truncated.

ND = number of thicknesses of conducting plane for which a solution is desired.

D = array containing thicknesses d of conducting plane for which a solution is desired.

The array IJOB is described by comment cards in the main program listing and is similar to that of Programs I and II. All lengths are in meters. The program is set up to obtain a solution for several values of d because only the matrix $[Y^b]$ need be computed for each new value of d. The half space matrices and excitation vectors are computed once then stored. The normalized wavenumbers and impedances of regions a, b, and c are calculated and if region b is lossy, the conductivity in siemens/meter and skin depth in meters

is obtained for the TE or the TM case, depending on IJOB(1) or IJOB(2), by calling subroutine SOLN3.

4.1.2 Listing of Main Program and Sample Output

```
C---------THICK SLIT PROGRAM THREE...
C         ANALYSIS OF SLIT WITH RECTANGULAR CROSS SECTION
C         MODAL SOLUTION FOR FILLED SLIT IN A GROUND
C         PLANE OF FINITE THICKNESS.
C
      COMPLEX CSQRT,ETANB,NMUB,NEPSB,WAVNB
      DIMENSION PHI(100),D(30),IJOB(8)
      COMMON /B/WAVNA,WAVNB,WAVNC,ETANA,ETANB,ETANC,WAVNO
      REAL NMUA,NMUC,NEPSA,NEPSC
      COMMON /GQI/A(10),T(10),NGQ
      DATA PI/3.141593/
100   FORMAT(6E11.4)
101   FORMAT(8I5)
102   FORMAT(2E20.7)
      READ(1,101) NGQ
      READ(1,102)(A(I),T(I),I=1,NGQ)
      WRITE(3,101) NGQ
      WRITE(3,102)(A(I),T(I),I=1,NGQ)
C---------IJOB(1) = 1... TE CASE
C         IJOB(2) = 1...TM CASE
C         IJOB(3) = NO. OF EXCITATIONS
C         IJOB(4) = 1...PRINT OUT CURRENTS
C         IJOB(5) = 1...COMP. TRANSMISSION COEFFS.
C         IJOB(6) = 1...COMP. GAIN AND FIELD PATTERNS
C         IJOB(7) = 1...COMP. PLANE WAVE APPROX. SOLUTION
C         IJOB(8) = 1...COMP. ZERO THICKNESS SOLUTION
      READ(1,101)(IJOB(I),I=1,8)
      WRITE(3,101)(IJOB(I),I=1,8)
      NEX=IJOB(3)
      K=1
      DO 1 I=1,NEX
      READ(1,100) PHI(K),PHI(K+1)
      WRITE(3,100) PHI(K),PHI(K+1)
      K=K+2
1     CONTINUE
      READ(1,100) NMUA,NEPSA
      READ(1,100) NMUB,NEPSB
      READ(1,100) NMUC,NEPSC
      READ(1,100) W,FMC,DPHI
      READ(1,101) N,NT,ND,NI
      WRITE(3,101) N,NT,ND,NI
```

```
      DO 3 I=1,ND
      READ(1,100) D(I)
      WRITE(3,100) D(I)
3     CONTINUE
      WAVNO=PI*FMC/150.
      WAVNA=SQRT(NMUA*NEPSA)
      WAVNB=CSQRT(NMUB*NEPSB)
      WAVNC=SQRT(NMUC*NEPSC)
      ETANA=SQRT(NMUA/NEPSA)
      ETANB=CSQRT(NMUB/NEPSB)
      ETANC=SQRT(NMUC/NEPSC)
200   FORMAT('1',15X,'THICK SLIT PARAMETERS...')
201   FORMAT('-',15X,'SLIT WIDTH W =',E11.4)
203   FORMAT('-',15X,'REGION PARAMETERS NORMALIZED BY THOSE OF',
     11X,'FREE SPACE')
204   FORMAT('-',15X,'REGION A',14X,'REGION C',14X,'REGIONB')
206   FORMAT('-','PERMEABILITY',3X,E11.4,11X,E11.4,11X,2E11.4)
207   FORMAT('-','PERMITTIVITY',3X,E11.4,11X,E11.4,11X,2E11.4)
208   FORMAT('-','IMPEDANCE',6X,E11.4,11X,E11.4,11X,2E11.4)
209   FORMAT('-','WAVENUMBER',5X,E11.4,11X,E11.4,11X,2E11.4)
210   FORMAT('-','CONDUCTIVITY OF REGION B =',E11.4,2X,'MHOS/METER')
211   FORMAT('-','SKIN DEPTH OF REGION B =',E11.4,2X,'METERS')
      WRITE(3,200)
      WRITE(3,201) W
      WRITE(3,203)
      WRITE(3,204)
      WRITE(3,206) NMUA,NMUC,NMUB
      WRITE(3,207) NEPSA,NEPSC,NEPSB
      WRITE(3,208) ETANA,ETANC,ETANB
      WRITE(3,209) WAVNA,WAVNC,WAVNB
      W=W*WAVNO
      DC=W/N
      IF(AIMAG(NEPSB).EQ.0.) GO TO 2
      SIG=-FMC*AIMAG(NEPSB)/(.18E 5)
      SKD=-1./(WAVNO*AIMAG(WAVNB))
      WRITE(3,210) SIG
      WRITE(3,211) SKD
2     CONTINUE
      IF(IJOB(1).NE.1) GO TO 10
      CALL SOLN3(1,IJOB,PHI,NEX,NI,DPHI,DC,D,W,NT,N,ND)
10    IF(IJCB(2).NE.1) GO TO 20
      CALL SOLN3(2,IJOB,PHI,NEX,NI,DPHI,DC,D,W,NT,N,ND)
20    CONTINUE
      STOP
      END
```

Sample output is presented for a slit of rectangular cross section where regions a, b, and c are free space. The excitation consists of a normally incident plane wave. Gain and field patterns are not computed.

The data cards used are shown in Table 7. The magnetic currents for the case when d = 0 are printed out first in the order in which they appear on Γ_1 as shown in Figs. 8 and 9 of Appendix A. The real part, imaginary part, and magnitude of each expansion function coefficient is given. For the cases when $d \neq 0$, the first N coefficients for Γ_1 (in this example N = 6) are printed out followed by the next N coefficients for Γ_2. In Program III, the aperture faces are Γ_1 and Γ_2 and these coefficients determine the heights of the expansion functions which are ordered on Γ_1 and Γ_2 according to Figs. 8 and 9 of Appendix A.

Table 7. Arrangement of data cards to produce sample output in Program III. $ sign is in column one.

```
$DATA
   4
.1739274E 0             -.4305682E 0
.1739274E 0              .4305682E 0
.3260725E 0             -.1699905E 0
.3260725E 0              .1699905E 0
   1     1     1     1     1     0     0     1
0.E 0       0.E 0
1.E 0       1.E 0
1.E 0       0.E 0       1.E 0        0.E 0
1.E 0       1.E 0
.1E 0       300.E 0      2.E 0
   6    10     7    40
.05E 0
.1E 0
.2E 0
.25E 0
.3E 0
.4E 0
.5E 0
$STOP
//
```

```
    4
      0.1739274E 00          -0.4305682E 00
      0.1739274E 00           0.4305682E 00
      0.3260725E 00          -0.1699905E 00
      0.3260725E 00           0.1699905E 00
    1      1      1      1      1      0      0      1
  0.0000E 00 0.0000E 00
    6   10     7   40
  0.5000E-01
  0.1000E 00
  0.2000E 00
  0.2500E 00
  0.3000E 00
  0.4000E 00
  0.5000E 00

  THICK SLIT PARAMETERS...

  SLIT WIDTH W = 0.1000E 00

  REGION PARAMETERS NORMALIZED BY THOSE OF FREE SPACE

                REGION A              REGION C              REGIONB

PERMEABILITY    0.1000E 01            0.1000E 01            0.1000E 01 0.0000E 00

PERMITTIVITY    0.1000E 01            0.1000E 01            0.1000E 01 0.0000E 00

IMPEDANCE       0.1000E 01            0.1000E 01            0.1000E 01 0.0000E 00

WAVENUMBER      0.1000E 01            0.1000E 01            0.1000E 01 0.0000E 00

                        ...TE CASE...

  ZERO THICKNESS CASE FOR INCIDENCE- 0.0000E 00 0.0000E 00
               1 0.2039E 01-0.2730E 01 0.3407E 01
               2 0.8327E 00-0.1008E 01 0.1307E 01
               3 0.8432E 00-0.1005E 01 0.1312E 01
               4 0.8432E 00-0.1005E 01 0.1312E 01
               5 0.8327E 00-0.1008E 01 0.1307E 01
               6 0.2039E 01-0.2730E 01 0.3407E 01

  SLIT IMPEDANCE =-0.6191E 00 0.7904E 00
  QUASI-STATIC IMPEDANCE =    -0.6197672E 00      0.7760662E 00
```

```
EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.1238E 01


MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00


        D = 0.5000E-01
            1 0.1001E 01-0.1631E 01 0.1914E 01
            2 0.6489E 00-0.9997E 00 0.1192E 01
            3 0.6441E 00-0.9795E 00 0.1172E 01
            4 0.6441E 00-0.9796E 00 0.1172E 01
            5 0.6489E 00-0.9997E 00 0.1192E 01
            6 0.1001E 01-0.1631E 01 0.1914E 01
            7-0.9723E 00 0.2075E 01 0.2291E 01
            8-0.6369E 00 0.1282E 01 0.1432E 01
            9-0.6326E 00 0.1265E 01 0.1414E 01
           10-0.6326E 00 0.1265E 01 0.1414E 01
           11-0.6369E 00 0.1282E 01 0.1432E 01
           12-0.9723E 00 0.2075E 01 0.2291E 01

        D = 0.5000E-01


                   ZS                                ZT
   0.3824E 00   -0.6017E 00   -0.3736E 00    0.7703E 00


EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.9033E 00


MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00


        D = 0.1000E 00
            1 0.6999E 00-0.1071E 01 0.1280E 01
            2 0.4563E 00-0.6639E 00 0.8056E 00
            3 0.4573E 00-0.6604E 00 0.8033E 00
            4 0.4573E 00-0.6604E 00 0.8033E 00
            5 0.4563E 00-0.6639E 00 0.8056E 00
            6 0.6999E 00-0.1071E 01 0.1280E 01
            7-0.5737E 00 0.2049E 01 0.2128E 01
            8-0.3911E 00 0.1282E 01 0.1340E 01
            9-0.3941E 00 0.1277E 01 0.1337E 01
           10-0.3941E 00 0.1277E 01 0.1337E 01
           11-0.3911E 00 0.1282E 01 0.1340E 01
           12-0.5737E 00 0.2049E 01 0.2128E 01

        D = 0.1000E 00
```

ZS			ZT
0.2689E 00	-0.3993E 00	-0.2265E 00	0.7680E 00

EXCITATIONS AND TRANSMISSION COEFFS.
0.0000E 00 0.0000E 00 0.7901E 00

MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00

D = 0.2000E 00

1	0.6926E 00	0.1326E 00	0.7052E 00
2	0.4343E 00	0.9363E-01	0.4443E 00
3	0.4331E 00	0.9481E-01	0.4433E 00
4	0.4331E 00	0.9480E-01	0.4433E 00
5	0.4343E 00	0.9363E-01	0.4443E 00
6	0.6926E 00	0.1326E 00	0.7052E 00
7	0.1239E 00	0.2333E 01	0.2336E 01
8	0.4390E-01	0.1471E 01	0.1472E 01
9	0.3916E-01	0.1468E 01	0.1469E 01
10	0.3916E-01	0.1468E 01	0.1469E 01
11	0.4390E-01	0.1471E 01	0.1472E 01
12	0.1239E 00	0.2333E 01	0.2336E 01

D = 0.2000E 00

ZS			ZT
0.2600E 00	0.5350E-01	0.3450E-01	0.8787E 00

EXCITATIONS AND TRANSMISSION COEFFS.
0.0000E 00 0.0000E 00 0.9532E 00

MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00

D = 0.2500E 00

1	0.1113E 01	0.8340E 00	0.1391E 01
2	0.6885E 00	0.5415E 00	0.8760E 00
3	0.6854E 00	0.5426E 00	0.8741E 00
4	0.6854E 00	0.5426E 00	0.8742E 00
5	0.6885E 00	0.5415E 00	0.8760E 00
6	0.1113E 01	0.8340E 00	0.1391E 01
7	0.7399E 00	0.2648E 01	0.2750E 01
8	0.4272E 00	0.1679E 01	0.1732E 01
9	0.4210E 00	0.1676E 01	0.1728E 01
10	0.4210E 00	0.1676E 01	0.1728E 01
11	0.4272E 00	0.1679E 01	0.1732E 01
12	0.7399E 00	0.2648E 01	0.2750E 01

D = 0.2500E 00

ZS			ZT
0.4144E 00	0.3197E 00	0.2647E 00	0.1001E 01

EXCITATIONS AND TRANSMISSION COEFFS.
0.0000E 00 0.0000E 00 0.1320E 01

MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00

D = 0.3000E 00

1	0.2183E 01	0.1397E 01	0.2591E 01
2	0.1354E 01	0.9116E 00	0.1632E 01
3	0.1348E 01	0.9140E 00	0.1629E 01
4	0.1348E 01	0.9140E 00	0.1629E 01
5	0.1354E 01	0.9116E 00	0.1632E 01
6	0.2183E 01	0.1397E 01	0.2591E 01
7	0.1951E 01	0.2851E 01	0.3455E 01
8	0.1187E 01	0.1824E 01	0.2176E 01
9	0.1179E 01	0.1824E 01	0.2172E 01
10	0.1179E 01	0.1824E 01	0.2172E 01
11	0.1187E 01	0.1824E 01	0.2176E 01
12	0.1951E 01	0.2851E 01	0.3455E 01

D = 0.3000E 00

ZS			ZT
0.8142E 00	0.5371E 00	0.7195E 00	0.1083E 01

EXCITATIONS AND TRANSMISSION COEFFS.
0.0000E 00 0.0000E 00 0.2084E 01

MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00

D = 0.4000E 00

1	0.4077E 01	-0.1439E 01	0.4324E 01
2	0.2589E 01	-0.8467E 00	0.2724E 01
3	0.2586E 01	-0.8367E 00	0.2718E 01
4	0.2586E 01	-0.8367E 00	0.2718E 01
5	0.2589E 01	-0.8467E 00	0.2724E 01
6	0.4077E 01	-0.1439E 01	0.4324E 01
7	0.4021E 01	-0.6805E 00	0.4078E 01
8	0.2542E 01	-0.3697E 00	0.2569E 01
9	0.2538E 01	-0.3609E 00	0.2564E 01
10	0.2538E 01	-0.3609E 00	0.2564E 01
11	0.2542E 01	-0.3697E 00	0.2569E 01
12	0.4021E 01	-0.6805E 00	0.4078E 01

```
D = 0.4000E 00

                 ZS                                  ZT
0.1542E 01   -0.5204E 00     0.1517E 01   -0.2352E 00

EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.2904E 01

MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00

   D = 0.5000E 00
      1 0.6675E 01-0.2164E 00 0.6678E 01
      2 0.7691E 00-0.4982E-01 0.7707E 00
      3 0.3854E 01-0.1823E-01 0.3854E 01
      4-0.1032E-04 0.6939E-02 0.6939E-02
      5 0.4623E 01-0.8961E-01 0.4624E 01
      6 0.3592E 01-0.1958E 00 0.3597E 01
      7 0.5133E 01-0.2022E 00 0.5137E 01
      8 0.3081E 01-0.7739E-01 0.3082E 01
      9 0.1542E 01-0.1839E-02 0.1542E 01
     10 0.1542E 01-0.1861E-02 0.1542E 01
     11 0.3081E 01-0.7735E-01 0.3082E 01
     12 0.5133E 01-0.2022E 00 0.5137E 01

   D = 0.5000E 00

                 ZS                                  ZT
0.1626E 01   -0.4691E-01     0.1626E 01   -0.4690E-01

EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.3251E 01

                  ...TM CASE...

ZERO THICKNESS CASE FOR INCIDENCE- 0.0000E 00 0.0000E 00
      1-0.9596E-02-0.2548E 00 0.2550E 00
      2-0.1150E-01-0.3048E 00 0.3051E 00
      3-0.1233E-01-0.3267E 00 0.3270E 00
      4-0.1150E-01-0.3048E 00 0.3051E 00
      5-0.9596E-02-0.2548E 00 0.2550E 00

SLIT POLARIZABILITY = 0.1918E 00-0.7231E-02
QUASI-STATIC POLARIZABILITY =     0.2218380E 00    -0.1957949E-01
```

```
EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.9086E-02

MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00

        D = 0.5000E-01
            1-0.3179E-02-0.1986E 00 0.1986E 00
            2-0.4005E-02-0.2479E 00 0.2479E 00
            3-0.4333E-02-0.2690E 00 0.2691E 00
            4-0.4006E-02-0.2507E 00 0.2507E 00
            5-0.3179E-02-0.2017E 00 0.2017E 00
            6 0.8520E-03-0.8091E-01 0.8092E-01
            7 0.1269E-02 0.3374E-01 0.3377E-01
            8 0.1427E-02 0.4757E-01 0.4759E-01
            9 0.1271E-02 0.4905E-01 0.4906E-01
           10 0.8562E-03 0.3221E-01 0.3222E-01

        D = 0.5000E-01

                  ZS                           ZT
  -0.1549E 00   0.2480E-02    0.4729E-03   0.6804E-02

EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.3003E-04

MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00

        D = 0.1000E 00
            1-0.3162E-02-0.2037E 00 0.2038E 00
            2-0.3959E-02-0.2546E 00 0.2546E 00
            3-0.4276E-02-0.2749E 00 0.2749E 00
            4-0.3959E-02-0.2547E 00 0.2547E 00
            5-0.3162E-02-0.2039E 00 0.2039E 00
            6-0.2465E-03-0.1135E 00 0.1135E 00
            7-0.2625E-03-0.1727E-01 0.1727E-01
            8-0.2693E-03-0.1024E-01 0.1025E-01
            9-0.2601E-03-0.1973E-02 0.1990E-02
           10-0.2423E-03-0.4198E-03 0.4847E-03

        D = 0.1000E 00

                  ZS                           ZT
  -0.1581E 00   0.2456E-02   -0.1067E-03  -0.1195E-01
```

```
EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.8964E-04


MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00


          D = 0.2000E 00
               1-0.3198E-02-0.2050E 00 0.2050E 00
               2-0.4009E-02-0.2564E 00 0.2565E 00
               3-0.4331E-02-0.2769E 00 0.2770E 00
               4-0.4009E-02-0.2564E 00 0.2565E 00
               5-0.3198E-02-0.2050E 00 0.2050E 00
               6-0.5341E-03-0.1220E 00 0.1220E 00
               7-0.6660E-03-0.3061E-01 0.3062E-01
               8-0.7177E-03-0.2545E-01 0.2546E-01
               9-0.6636E-03-0.1531E-01 0.1532E-01
              10-0.5299E-03-0.8848E-02 0.8863E-02


          D = 0.2000E 00


                    ZS                         ZT
   -0.1591E 00    0.2486E-02   -0.2593E-03   -0.1685E-01


EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.1777E-03


MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00


          D = 0.2500E 00
               1-0.3200E-02-0.2050E 00 0.2050E 00
               2-0.4011E-02-0.2565E 00 0.2566E 00
               3-0.4334E-02-0.2770E 00 0.2771E 00
               4-0.4011E-02-0.2565E 00 0.2566E 00
               5-0.3200E-02-0.2050E 00 0.2050E 00
               6-0.5450E-03-0.1223E 00 0.1223E 00
               7-0.6814E-03-0.3111E-01 0.3112E-01
               8-0.7348E-03-0.2603E-01 0.2604E-01
               9-0.6790E-03-0.1582E-01 0.1583E-01
              10-0.5408E-03-0.9167E-02 0.9183E-02


          D = 0.2500E 00


                    ZS                         ZT
   -0.1592E 00    0.2488E-02   -0.2651E-03   -0.1703E-01
```

```
EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.1816E-03


MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00


     D = 0.3000E 00
        1-0.3200E-02-0.2050E 00 0.2050E 00
        2-0.4012E-02-0.2565E 00 0.2566E 00
        3-0.4334E-02-0.2770E 00 0.2771E 00
        4-0.4012E-02-0.2565E 00 0.2566E 00
        5-0.3200E-02-0.2050E 00 0.2050E 00
        6-0.5473E-03-0.1223E 00 0.1224E 00
        7-0.6847E-03-0.3122E-01 0.3123E-01
        8-0.7384E-03-0.2615E-01 0.2616E-01
        9-0.6823E-03-0.1592E-01 0.1594E-01
       10-0.5432E-03-0.9236E-02 0.9252E-02


     D = 0.3000E 00


               ZS                         ZT
  -0.1592E 00   0.2488E-02   -0.2663E-03  -0.1707E-01


EXCITATIONS AND TRANSMISSION COEFFS.
 0.0000E 00 0.0000E 00 0.1824E-03


MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00


     D = 0.4000E 00
        1-0.3200E-02-0.2050E 00 0.2050E 00
        2-0.4012E-02-0.2565E 00 0.2566E 00
        3-0.4335E-02-0.2770E 00 0.2771E 00
        4-0.4012E-02-0.2565E 00 0.2566E 00
        5-0.3200E-02-0.2050E 00 0.2050E 00
        6-0.5480E-03-0.1224E 00 0.1224E 00
        7-0.6855E-03-0.3125E-01 0.3126E-01
        8-0.7394E-03-0.2619E-01 0.2620E-01
        9-0.6832E-03-0.1595E-01 0.1597E-01
       10-0.5438E-03-0.9254E-02 0.9270E-02


     D = 0.4000E 00


               ZS                         ZT
  -0.1592E 00   0.2488E-02   -0.2667E-03  -0.1708E-01
```

EXCITATIONS AND TRANSMISSION COEFFS.
0.0000E 00 0.0000E 00 0.1827E-03

MODAL MAGNETIC CURRENTS... 0.0000E 00 0.0000E 00

D = 0.5000E 00
1-0.3200E-02-0.2050E 00 0.2050E 00
2-0.4012E-02-0.2565E 00 0.2566E 00
3-0.4335E-02-0.2770E 00 0.2771E 00
4-0.4012E-02-0.2565E 00 0.2566E 00
5-0.3200E-02-0.2050E 00 0.2050E 00
6-0.5480E-03-0.1224E 00 0.1224E 00
7-0.6856E-03-0.3125E-01 0.3126E-01
8-0.7394E-03-0.2619E-01 0.2620E-01
9-0.6832E-03-0.1595E-01 0.1597E-01
10-0.5438E-03-0.9255E-02 0.9271E-02

D = 0.5000E 00

	ZS		ZT
-0.1592E 00	0.2488E-02	-0.2667E-03	-0.1708E-01

EXCITATIONS AND TRANSMISSION COEFFS.
0.0000E 00 0.0000E 00 0.1827E-03

4.1.3 Minimum Storage Requirements for Arrays

The minimum storage required for each array used in Program III is given here. Arrays which are execution-time dimensioned are not mentioned (i.e., array C in subroutine CSMTZ). Integers which are used to specify the array sizes are defined by:

NEX = number of excitations or sources in region a to be considered.

ND = number of thicknesses, d, of conducting plane to be computed for a given aperture width w.

NGQ = order of Gaussian quadrature formula used for numerical integration.

N = number of subsections into which each aperture face is divided.

NT = number of terms at which infinite summations are truncated.

NI = number of points in region c at which far field is computed to obtain field patterns.

The allocations required for the arrays in common blocks are given by:

```
GQI:  DIMENSION A(NGQ), T(NGQ)
```

The allocations required for arrays used in the subroutines are as follows:

```
MAIN PGM:  DIMENSION PHI(2*NEX), D(ND),
           IJOB(8)

SOLN3:     COMPLEX Y(N*(2*N+1)), XI(2*N*NEX),
           YHSA(N), YHSC(N), YHS(N),
           XD(2*N*NEX), XZ(N), XIPW(2*N*NEX),
           YZ(N)
           DIMENSION PHI(2*NEX), PT(NEX),
           PIN(NEX), D(ND), IJOB(8)
```

```
        CSUM:  COMPLEX XI(2*N*NEX)
         PWB:  COMPLEX YHSA(N), YHSC(N), XI(2*N*NEX),
               XIPW(2*N*NEX), Y(N*(N+1)/2),
               YDA(N), YDC(N)
               DIMENSION PHI(2*NEX)
      TRANS2:  COMPLEX YHS(N), YAUX(N), VC(N),
               VM(2*N*NEX)
               DIMENSION PHI(2*NEX), PIN(NEX),
               PT(NEX)
       GAIN2:  COMPLEX XI(2*N*NEX)
               DIMENSION PHI(2*NEX), FP(NI),
               FT(NI), GA(NI), PT(NEX)
          YB:  COMPLEX Y(N*(2*N+1)), ST1(NT),
               ST2(NT)
               DIMENSION SIN2(NT)
   EXTE2 and
      EXTM2:   COMPLEX XI(2*N*NEX)
               DIMENSION PHI(2*NEX), PIN(NEX)
       GELS:   COMPLEX R(2*N*NEX), AUX(N*(2*N+1))
      CSMTZ:   COMPLEX E(N), ES(N)
```

4.2 Subroutine YB

This subroutine forms the upper right triangular portion of the matrix $[Y^b]$ given by Eq. (7-2). A specialization of Eqs. (6-24) and (6-26) is used for the TE case and a specialization of Eqs. (6-36) and (6-38) is used for the TM case. A finite number, NT, of terms is taken in the infinite summations. The integer NU equals the number of magnetic current expansion functions on the aperture faces. The

computations are quite straightforward and the result is stored in the array Y. The argument parameters are defined by

Input: N = number of subsections into which each aperture face is divided.

DCK = subsection length W/N times k_b.

WK = $k_b w$

DK = $k_b d$

ET = η_b/η_o

WAV = k_b/k_o

NT = number at which infinite summations are truncated.

ID = integer option variable, ID = 1 for TE case and ID = 2 for TM case.

Output: Y = array containing elements of the upper right triangular portion of the matrix $[Y^b]$ of Eq. (7-2).

```
      SUBROUTINE YB(Y,N,DCK,WK,DK,ET,WAV,NT,ID)
      COMPLEX Y(2000),ST1(150),ST2(150)
      COMPLEX CSIN,CCOS,CSQRT,U,YB11,YB12,CXP,KBD
      COMPLEX ET,WAV,DK,WK,DCK,CONST,YS11,YS12
      DIMENSION SIN2(150)
      DATA U/(0.,1.)/,PI/3.141593/
C--------ID = 1 MEANS TE CASE...OTHERWISE TM CASE
      N2=2*N
      NU=N+1-ID
      CONST=DCK/(N*U*WAV*ET)
      IF(ID.NE.1) GO TO 3
      IF(ABS(AIMAG(DK)).GE.50.) GO TO 1
      YB12=CONST/CSIN(DK)
      YB11=YB12*CCOS(DK)
      GO TO 2
1     MF=1
      IF(AIMAG(DK).GT.0.) MF=-1
      YB11=MF*CONST*U
      YB12=0.
2     CONTINUE
```

```
3       DO 5 IP=1,NT
        CXP=CSQRT(1.-(IP*PI/WK)**2)
        KBD=CXP*DK
        IF(ABS(AIMAG(KBD)).GE.50.) GO TO 6
        IF(ID.NE.1) GO TO 7
        ST2(IP)=1./(CSIN(KBD)*CXP*(IP*PI/N2)**2)
        ST1(IP)=ST2(IP)*CCOS(KBD)
        GO TO 10
7       ST2(IP)=CXP/(CSIN(KBD)*(IP*PI/N2)**4)
        ST1(IP)=ST2(IP)*CCOS(KBD)
        GO TO 10
6       MF=1
        IF(AIMAG(KBD).GT.0.) MF=-1
        ST2(IP)=0.
        IF(ID.NE.1) GO TO 8
        ST1(IP)=MF*U/(CXP*(IP*PI/N2)**2)
        GO TO 10
8       ST1(IP)=MF*U*CXP/(IP*PI/N2)**4
10      CONTINUE
5       CONTINUE
        CONST=CONST*2.
        DO 11 IP=1,NT
        SIN2(IP)=(SIN(IP*PI/N2))**2
11      CONTINUE
        M=NU*(NU+1)/2
        K=1
        DO 15 I=1,NU
        IMN=M+(I-1)*NU
        DO 15 J=1,I
        YS11=0.
        YS12=0.
        IF(ID.NE.1) GO TO 17
        DO 16 IP=1,NT
        CS=COS(IP*PI*(I-.5)/N)*COS(IP*PI*(J-.5)/N)
        YS11=YS11+ST1(IP)*CS*SIN2(IP)
        YS12=YS12+ST2(IP)*CS*SIN2(IP)
16      CONTINUE
        Y(K)=YB11+YS11*CONST
        Y(IMN+K)=YB12+YS12*CONST
        K=K+1
        GO TO 18
17      DO 19 IP=1,NT
        SN=SIN(I*IP*PI/N)*SIN(J*IP*PI/N)
        YS11=YS11+ST1(IP)*SN*(SIN2(IP))**2
        YS12=YS12+ST2(IP)*SN*(SIN2(IP))**2
19      CONTINUE
        Y(K)=YS11*CONST
        Y(IMN+K)=YS12*CONST
        K=K+1
18      CONTINUE
15      CONTINUE
        M1=M
        N1=NU-1
        K=2
```

```
      DO 21 I=1,N1
      M2=M1+NU+I
      DO 20 J=K,NU
      Y(M1+J)=Y(M2+I)
      M2=M2+NU+J
20    CONTINUE
      K=K+1
      M1=M1+NU+I
21    CONTINUE
      K=1
      DO 22 I=1,NU
      IMN=M+I*NU
      DO 22 J=1,I
      Y(IMN+K)=Y(K)
      K=K+1
22    CONTINUE
      RETURN
      END
```

4.3 Subroutine CSUM

This subroutine computes the slit impedance, defined by Eq. (5-28) for the TE case, given by variable ZT. A slit polarizability, defined by Eq. (5-34), is computed for the TM case and represented by ZT. Another quantity, which is obtained by replacing $\underline{M}^3(y')$ in Eqs.(5-28) and (5-34) by $\underline{M}^1(y')$, is computed and represented by the variable ZS. This is also an impedance for the TE case and a polarizability for the TM case. These quantities are printed out at execution. The argument parameters are defined by

Input: ID = integer option variable. ID = 1 for TE case and ID = 2 for the TM case.

XI = array containing solution vectors to Eq. (7-1) for each excitation.

NU = number of elements in vectors $\vec{V}^1$ and $\vec{V}^2$ of Eq. (7-1).

NEX = number of excitations.

N = number of subsections into which each aperture face is divided.

$W = k_o w$.

```
      SUBROUTINE CSUM(ID,XI,NU,NEX,N,W)
      COMPLEX XI(1000),ZS,ZT,U,C1
      DATA U/(0.,1.)/
      C1=1.
      IF(ID.EQ.2) C1=1./(U*W)
      WRITE(3,100)
      K=1
      DO 1 II=1,NEX
      ZS=0.
      DO 2 I=1,NU
      ZS=ZS+XI(K)
      K=K+1
2     CONTINUE
      ZS=ZS*C1/(2*N)
      ZT=0.
      DO 3 I=1,NU
      ZT=ZT+XI(K)
      K=K+1
3     CONTINUE
      ZT=ZT/(2*N)
      WRITE(3,101) ZS,ZT
1     CONTINUE
100   FORMAT('-',15X,'ZS',25X,'ZT')
101   FORMAT(' ',4E13.4)
      RETURN
      END
```

4.4 Subroutine PWB

This subroutine forms the system of Eqs. (7-1) when the plane wave approximation to the fields in the slit cross section is used. Equations (7-9) and (7-10) are used to compute $[Y^b]$ for the TE case and Eqs. (7-15) and (7-16) are used to compute $[Y^b]$ for the TM case. The argument parameters are defined by:

Input: YHSA = first column of $[Y^a]$.

YHSC = first column of $[Y^c]$.

XI = array containing right hand side vectors of Eq. (7-1).

N = number of subsections into which each aperture face is divided.

NEX = number of excitations or sources in region a to be considered.

ID = integer option variable, ID = 1 for TE case and ID = 2 for TM case.

DK = $k_b d$

DY = $k_o w/N$

ETB = η_b/η_o

PHI = excitation array defined in Section 2.1.1.

XIPW = array containing solution vectors of Eq. (7-1).

JWR = integer option variable, if JWR = 1 magnetic currents are printed out and if JWR $\neq$ 1, they are not printed out.

The magnetic currents are computed and returned in array XIPW for use by subroutines TRANS2 or GAIN2.

```
      SUBROUTINE PWB(YHSA,YHSC,XI,N,NEX,ID,DK,DY,ETB,PHI,XIPW,JWR)
      COMPLEX YHSA(40),YHSC(40),XI(1000),XIPW(1000)
      COMPLEX Y(4000),YDA(40),YDC(40),C1,C2
      COMPLEX DK,ETB,CSIN,CCOS,U
      DIMENSION PHI(100)
      DATA U/(0.,1.)/
      NU=N-ID+1
      M=NU*(NU+1)/2
      IF(AIMAG(DK).GE.50.) GO TO 1
      C2=U*DY/(CSIN(DK)*ETB)
      C1=-C2*CCOS(DK)
      GO TO 2
1     MF=-1
      IF(AIMAG(DK).GT.0.) MF=1
      C1=MF*DY/ETB
      C2=0.
2     CONTINUE
      YDA(1)=YHSA(1)+C1*(4-ID)/3.
      YDA(2)=YHSA(2)+C1*(ID-1)/6.
      YDC(1)=YHSC(1)+C1*(4-ID)/3.
      YDC(2)=YHSC(2)+C1*(ID-1)/6.
      DO 3 I=3,NU
      YDA(I)=YHSA(I)
      YDC(I)=YHSC(I)
3     CONTINUE
      K=1
      DO 4 J=1,NU
      KM=M+J*NU
      DO 4 I=1,J
      Y(K)=YDA(J-I+1)
      Y(KM+K)=YDC(J-I+1)
      K=K+1
4     CONTINUE
      K=M+1
      DO 8 J=1,NU
      DO 5 I=1,NU
      IF(IABS(I-J).GT.1) GO TO 6
      IF(IABS(I-J).EQ.1) Y(K)=C2*(ID-1)/6.
      IF(I.EQ.J) Y(K)=C2*(4-ID)/3.
      GO TO 7
6     Y(K)=0.
7     K=K+1
5     CONTINUE
      K=K+J
8     CONTINUE
      NX=NU*NEX*2
      DO 10 I=1,NX
      XIPW(I)=XI(I)
10    CONTINUE
      N2=2*NU
      M2=N2*(N2+1)/2
      CALL GELS(XIPW,Y,N2,NEX,M2)
100   FORMAT('1',15X,'MAGNETIC CURRENT USING PLANE WAVE ASSUMPTION'
     1,2X,'PHI =',2E11.4)
```

```
101    FORMAT(I5,3E17.7)
102    FORMAT('-',1X,'POS.',6X,'REAL',13X,'IMAG',13X,'MAG',//)
103    FORMAT('-',10X,'APERTURE FACE',I5,3X,'...',//)
       IF(JWR.NE.1) RETURN
       K=1
       DO 11 J=1,NEX
       WRITE(3,100) PHI(2*J-1),PHI(2*J)
       WRITE(3,102)
       DO 11 I=1,2
       WRITE(3,103) I
       DO 11 II=1,NU
       VA=CABS(XIPW(K))
       WRITE(3,101) II,XIPW(K),VA
       K=K+1
11     CONTINUE
       RETURN
       END
```

4.5 Subroutine SOLN3

This subroutine forms and solves the system of equations (7-1) when the various approximate solutions are used. First the zero thickness (d = 0) slit is solved for comparison purposes. Next the slit impedance, ZQS, using Eq. (8-7), is computed for the TE case and also a slit polarizability SP is computed for the TM case based on a similar quasi-static formula. A modal solution, using NT terms in the infinite summations, is always computed for comparison purposes for all values of conducting plane thickness d. CSUM is used to compute the impedance and polarizabilities from the modal results. PWB is used to obtain a solution using the plane wave approximation which is usually compared directly to the modal solution. The integer array IJOB is used to avoid computations and printout which are not desired. The argument parameters are defined by:

Input: ID = integer variable, ID = 1 for TE case and ID = 2 for TM case.

IJOB = integer option array defined by comment cards in the main program.

PHI = excitation array defined in Section 2.1.1.

NEX = number of excitations or sources in region a.

NI = number of points in region c at which far field is computed to obtain field patterns.

DPHI = increment in degrees at which field patterns are computed.

DC = $k_o w/N$

D = array containing thicknesses $k_o d$ of conducting plane.

W = aperture width $k_o w$

NT = number at which infinite summations are truncated.

N = number of subsections into which each aperture face is divided.

ND = number of thicknesses d to be considered.

```
SUBROUTINE SCLN3(ID,IJOB,PHI,NEX,NI,DPHI,DC,D,W,NT,N,ND)
COMPLEX Y(2500),XI(3000),YHSA(40),YHSC(40),YHS(40)
COMPLEX XD(3000),XZ(40),C1,KB,ETB,ZQS,SP,DK
COMPLEX XIPW(3000),YZ(40),U
DIMENSION PHI(100),PT(100),PIN(100),D(30),IJOB(8)
REAL KA,KC,KO
COMMON /B/KA,KB,KC,ETA,ETB,ETC,KO
DATA PI/3.141593/,GAM/1.78107/,U/(0.,1.)/
```

```
      NU=N+1-ID
      M=NU*(NU+1)/2
      N2=2*NU
      M2=N2*(N2+1)/2
      NX=N2*NEX
C--------FORM EXCITATION AND HALF SPACE MATRICES
      IF(ID.NE.1) GO TO 1
      CALL YHSTE(YHSA,NU,DC*KA,KA,ETA)
      CALL YHSTE(YHSC,NU,DC*KC,KC,ETC)
      CALL EXTE2(XI,N,NEX,PHI,DC*KA,W*KA,ETA,KA,N2,PIN,KO)
      WRITE(3,105)
      GO TO 2
1     CALL YHSTM(YHSA,NU,DC*KA,KA,ETA)
      CALL YHSTM(YHSC,NU,DC*KC,KC,ETC)
      CALL EXTM2(XI,N,NEX,PHI,DC*KA,W*KA,ETA,KA,N2,PIN,KO)
      WRITE(3,106)
2     IF(IJOB(8).NE.1) GO TO 20
C--------SOLVE ZERO THICKNESS CASE
      C1=YHSA(1)+YHSC(1)
      DO 4 I=1,NU
      YHS(I)=(YHSA(I)+YHSC(I))/C1
4     CONTINUE
      K1=1
      K2=NU+1
      DO 5 J=1,NEX
      DO 6 I=1,NU
      XZ(I)=XI(K1)/C1
      K1=K1+1
6     CONTINUE
      K1=K1+NU
      CALL CSMTZ(YHS,YZ,XZ,NU)
      WRITE(3,100) PHI(2*J-1),PHI(2*J)
      ZQS=0.
      DO 7 I=1,NU
      CM=CABS(YZ(I))
      XD(K2)=-YZ(I)
      IF(IJOB(4).EQ.1) WRITE(3,101) I,YZ(I),CM
      K2=K2+1
      ZQS=ZQS-YZ(I)
7     CONTINUE
      K2=K2+NU
      IF(ID.EQ.1) GO TO 3
      SP=ZQS/(N*2*U*W)
      WRITE(3,102) SP
      GO TO 9
3     ZQS=ZQS/(2*N)
      WRITE(3,103) ZQS
9     CONTINUE
5     CONTINUE
      IF(ID.NE.1) GO TO 8
      ZQS=-PI/(W*(PI-2.*U*ALOG(GAM*W/8.)))
      WRITE(3,108) ZQS
      GO TO 10
8     SP=PI/(16.*(1.+((W/4.)**2)*(2.*ALOG(GAM*W/8.)-1.+U*PI)))
      WRITE(3,109) SP
10    CONTINUE
```

```
      IF(IJOB(5).EQ.1) CALL TRANS2(YHSC,PIN,PHI,NEX,XD,ID,PT,NU)
      IF(IJOB(6).EQ.1) CALL GAIN2(XD,PHI,DPHI,NI,PT,ID,NEX,NU,W,DC)
20    CONTINUE
C--------COMP. MODAL SCLN. FOR EACH THICKNESS
      DO 11 II=1,ND
      DO 12 I=1,NX
      XD(I)=XI(I)
12    CONTINUE
      DK=K0*KB*D(II)
      CALL YB(Y,N,DC*KB,W*KB,DK,ETB,KB,NT,ID)
      K=1
      DO 13 J=1,NU
      MJ=M+J*NU
      DO 13 I=1,J
      Y(K)=Y(K)+YHSA(J-I+1)
      Y(MJ+K)=Y(MJ+K)+YHSC(J-I+1)
      K=K+1
13    CONTINUE
      CALL GELS(XD,Y,N2,NEX,M2)
      IF(IJOB(4).NE.1) GO TO 17
      K=1
      DO 14 J=1,NEX
      WRITE(3,104) PHI(2*J-1),PHI(2*J)
      WRITE(3,107) D(II)
      DO 14 I=1,N2
      CM=CABS(XD(K))
      WRITE(3,101) I,XD(K),CM
      K=K+1
14    CONTINUE
17    WRITE(3,107) D(II)
      CALL CSUM(ID,XD,NU,NEX,N,W)
      IF(IJOB(5).EQ.1) CALL TRANS2(YHSC,PIN,PHI,NEX,XD,ID,PT,NU)
      IF(IJOB(6).EQ.1) CALL GAIN2(XD,PHI,DPHI,NI,PT,ID,NEX,NU,W,DC)
      IF(IJOB(7).NE.1) GO TO 15
      CALL PWB(YHSA,YHSC,XI,N,NEX,ID,DK,DC,ETB,PHI,XIPW,IJOB(4))
      IF(IJOB(5).EQ.1) CALL TRANS2(YHSC,PIN,PHI,NEX,XIPW,ID,PT,NU)
15    CONTINUE
11    CONTINUE
100   FORMAT('-','ZERO THICKNESS CASE FOR INCIDENCE-',2E11.4)
101   FORMAT(' ',I10,3E11.4)
102   FORMAT('-','SLIT POLARIZABILITY =',2E11.4)
103   FORMAT('-','SLIT IMPEDANCE =',2E11.4)
104   FORMAT('-','MODAL MAGNETIC CURRENTS...',2E11.4)
105   FORMAT('1',20X,'...TE CASE...')
106   FORMAT('1',20X,'...TM CASE...')
107   FORMAT('-',5X,' D =',E11.4)
108   FORMAT(' ','QUASI-STATIC IMPEDANCE =',2E17.7)
109   FORMAT(' ','QUASI-STATIC POLARIZABILITY =',2E17.7)
      RETURN
      END
```

4.6 Subroutines TRANS2 and GAIN2

These subroutines are essentially the same as those described in Section 3.9 except that several arrays used in Section 3.9 are eliminated for their use in Program III. They are simply listed here to avoid confusion.

```
      SUBROUTINE TRANS2(YHS,PIN,PHI,NEX,VM,ID,PT,NU)
      COMPLEX YHS(40),YAUX(40),VC(40),VM(1000)
      COMPLEX CONJG,SUM,S
      DIMENSION PHI(100),PIN(100),PT(100)
      DATA PI/3.141593/
100   FORMAT('-','EXCITATIONS AND TRANSMISSION COEFFS.')
101   FORMAT(' ',3E11.4)
      NC=NU
      NR=2*NU
      DO 1 I=1,NC
      YAUX(I)=CONJG(YHS(I))
1     CONTINUE
      WRITE(3,100)
      K=NU
      DO 10 J=1,NEX
      DO 3 I=1,NC
      VC(I)=CONJG(VM(K+I))
3     CONTINUE
      S=0.
      DO 4 I=1,NC
      S=S+VM(K+I)*VC(I)
4     CONTINUE
      SUM=YAUX(1)*S
      DO 5 L=2,NC
      S=0.
      NLI=NC-L+1
      DO 6 I=1,NLI
      S=S+VM(K+I)*VC(L+I-1)+VM(K+L+I-1)*VC(I)
6     CONTINUE
      SUM=SUM+YAUX(L)*S
5     CONTINUE
      PT(J)=REAL(SUM)
      T=PT(J)/PIN(J)
      WRITE(3,101) PHI(2*J-1),PHI(2*J),T
      K=K+NR
10    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GAIN2(XI,PHI,DPHI,NI,PT,ID,NEX,NU,W,DC)
      COMMON /B/WAVNA,WAVNB,WAVNC,ETANA,ETANB,ETANC,WAVNO
      COMPLEX XI(1000),U,CEXP,S,YM,ETANB,WAVNB
      DIMENSION PHI(100),FP(100),FT(100),GA(100),PT(100)
      DATA PHO/-1.570796/,U/(0.,1.)/,ETAO/377./
100   FORMAT('1','TE GAIN AND FIELD PATTERNS')
101   FORMAT('1','TM GAIN AND FIELD PATTERNS')
102   FORMAT(' ',3E15.4)
      NR=2*NU
      WK2=W*WAVNC/2.
      D2=DC*WAVNC/2.
      DPH=-DPHI*PHO/90.
      PH=PHO
      CPH=-1.
      DO 2 I=1,NI
      SN=SIN(PH)
      IF(ID.EQ.2) CPH=-COS(PH)
      IF(SN.EQ.0) GO TO 3
      FT(I)=-SIN(D2*SN)/(D2*SN)
      IF(ID.EQ.2) FT(I)=CPH*FT(I)**2
      GO TO 4
3     FT(I)=CPH
4     CONTINUE
      PH=PH+DPH
2     CONTINUE
      K=NU
      DO 10 J=1,NEX
      PH=PHO
      DO 6 I=1,NI
      SN=SIN(PH)
      S=0.
      DO 7 L=1,NU
      YM=((2*L+ID-2)*D2-WK2)*SN*U
      S=S+XI(K+L)*CEXP(YM)*FT(I)
7     CONTINUE
      F=2.*D2*CABS(S)
      FP(I)=F
      GA(I)=F*F/(2.*WAVNC*ETANC*PT(J))
      PH=PH+DPH
6     CONTINUE
      FM=FP(1)
      DO 8 I=2,NI
      IF(FM.LT.FP(I)) FM=FP(I)
8     CONTINUE
      DO 9 I=1,NI
      FP(I)=FP(I)/FM
9     CONTINUE
      PH=-90.
      IF(ID.EQ.1) WRITE(3,100)
      IF(ID.EQ.2) WRITE(3,101)
      DO 5 I=1,NI
      WRITE(3,102) PH,GA(I),FP(I)
      PH=PH+DPHI
5     CONTINUE
      K=K+NR
10    CONTINUE
      RETURN
      END
```

4.7 Subroutine CSMTZ

This subroutine solves the set of equations

$$L_N s_N = d_N$$

where L_N is an $N \times N$ symmetric Toeplitz matrix. The argument parameters are defined by

Input: C = array whose elements are the first row of L_N.

D = array whose elements are those of d_N.

N = number of unknowns.

Output: S = array containing the solution vector s_N.

It is assumed that the above matrix equation is normalized so that the first element of the first row of L_N is equal to unity. The algorithm used is explained in great detail in [6] and [7] and a derivation will not be repeated here. An explanation of the subroutine logic, however, will be reviewed using the notation developed in [6] and [7].

Notation:

Capital letter B_j indicates a $j \times j$ matrix.

Lower case letter b_j indicates a $j \times 1$ matrix.

~ denotes transpose

$\hat{a}_i$ is a column vector made up of the first i components in reversed order.

Greek letters indicate scalars.

Capital letter C(j) indicates the jth component of array C.

The matrix L_N is bordered as follows:

$$L_N = \begin{bmatrix} 1 & \tilde{r}_{N-1} \\ r_{N-1} & L_{N-1} \end{bmatrix}$$

where $\tilde{r}_{N-1} = (C(2), C(3), \ldots, C(N))$. The algorithm is based on a recursion relation with initial values given by

$$s_1 = \delta_1, \ \rho_1 = -C(2), \qquad \lambda_1 = 1 - C(2)*C(2)$$

Recursion of s_i, $\hat{e}_i$, and λ_i for $i = 1,2,\ldots,N-2$ is given by:

$$\theta_i = \delta_{i+1} - \tilde{s}_i \hat{r}_i$$

$$\eta_i = - C(i+2) - \tilde{r}_i \hat{e}_i$$

$$s_{i+1} = \begin{bmatrix} s_i + (\theta_i/\lambda_i)\hat{e}_i \\ \theta_i/\lambda_i \end{bmatrix}$$

$$e_{i+1} = \begin{bmatrix} e_i + (\eta_i/\lambda_i)\hat{e}_i \\ \eta_i/\lambda_i \end{bmatrix}$$

$$\lambda_{i+1} = \lambda_i - \eta_i^2/\lambda_i$$

The last computed values are θ_{N-1}, η_{N-2}, s_N, e_{N-1}, and λ_{N-1}. Note that $\tilde{a}_N$, g_N, and γ appearing in [6] are not needed here because of symmetry.

The subroutine computations are then carried out as follows:

Compute initial values s_1, e_1, λ_1

DO 1 i = 1,N-2

DO 2 j = 1,i

Compute S1 = $\tilde{s}_i \hat{r}_i$ and E1 = $\tilde{r}_i \hat{e}_i$

2 Continue

Compute η_i, θ_i

DO 3 k = 1, i

Compute s_{i+1}

Store e_i in dummy array ES because e_i is needed in DO loop 6 to recompute itself.

3 Continue

DO 6 k = 1, i

Compute e_i

6 Continue

Compute λ_{i+1}

1 Continue

Compute s_N

Return

End

```
SUBROUTINE CSMTZ(C,S,D,N)
COMPLEX C(N),S(N),D(N)
COMPLEX E(100),ES(100)
COMPLEX LA,E1,S1,ET,ET1,TH,TH1
S(1) = D(1)
E(1) = -C(2)
LA=1.-C(2)*C(2)
N1=N-2
N2=N-1
DO 1 I=1,N1
E1=(0.,0.)
S1=(0.,0.)
```

```
      DO 2 J=1,I
      S1=S1+S(J)*C(I-J+2)
      E1=E1+E(I-J+1)*C(J+1)
2     CONTINUE
      ET=-C(I+2)-E1
      TH=D(I+1)-S1
      TH1=TH/LA
      ET1=ET/LA
      DO 3 K=1,I
      S(K)=S(K)+TH1*E(I-K+1)
      ES(K)=E(K)
3     CONTINUE
      DO 6 K=1,I
      E(K)=ES(K)+ET1*ES(I-K+1)
6     CONTINUE
      S(I+1)=TH/LA
      E(I+1)=ET1
      LA=LA-ET*ET1
1     CONTINUE
      S1=(0.,0.)
      DO 4 J=1,N2
      S1=S1+S(J)*C(N-J+1)
4     CONTINUE
      TH=D(N)-S1
      TH1=TH/LA
      DO 5 K=1,N2
      S(K)=S(K)+TH1*E(N-K)
5     CONTINUE
      S(N2+1)=TH1
      RETURN
      END
```

Appendix A

INTERPRETATION AND COMPARISON OF MODAL AND NON-MODAL RESULTS

The unknowns in the non-modal solution consist of magnetic currents on the slit faces adjacent to the half space regions and electric currents which exist on the entire contour which defines the slit cross section. These quantities are represented by sets of expansion functions, the coefficients of which are obtained by solving a set of simultaneous linear equations (2-17) for the vectors $\vec{V}^1$, $\vec{V}^3$, and $\eta_o \vec{I}$. The expansion functions used are different in the two polarizations and their placement on the slit cross section is shown in Figs. 6 and 7. Thus, the tangential electric and magnetic fields at the two slit faces comes out directly in this method of solution.

The unknowns in the modal solution consist of magnetic currents on the two slit faces adjacent to the half space regions as well as each aperture coupling two rectangular sub-regions. These quantities are represented by sets of expansion functions, the coefficients of which are determined by solving a set of simultaneous linear equations (6-8) for vectors $\vec{V}_1$, $\vec{V}_2, \ldots,$ $\vec{V}_{NB+1}$. The expansion functions used are again different in the two polarizations and their placement on a typical slit cross section is shown in Figs. 8 and 9. An extra computational effort is required here in order to obtain the tangential magnetic field at points on the contour defining the slit cross section. This is outlined in Appendix C.

The quantities which may be directly compared, then, are the magnetic currents on the aperture faces adjacent to the half space regions.

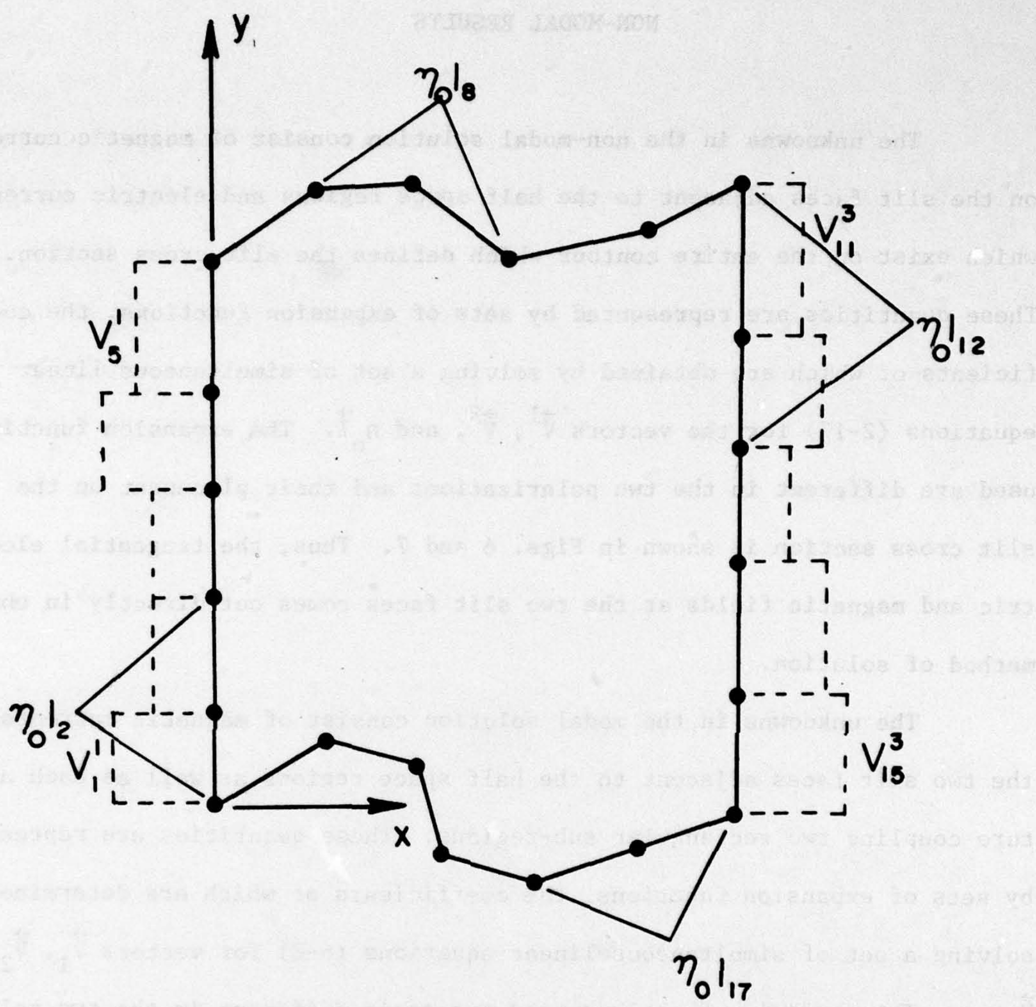

Fig. 6. Placement of some typical elements of TE solution vector on contour C for Program I. Dashed lines represent $\hat{z}$ directed expansion functions for $\vec{V}^1$ and $\vec{V}^3$ and solid lines represent $\hat{t}$ directed expansion functions for $\eta_o\vec{I}$, ND = 1,6,11,16,22. The nodes are numbered clockwise from the origin.

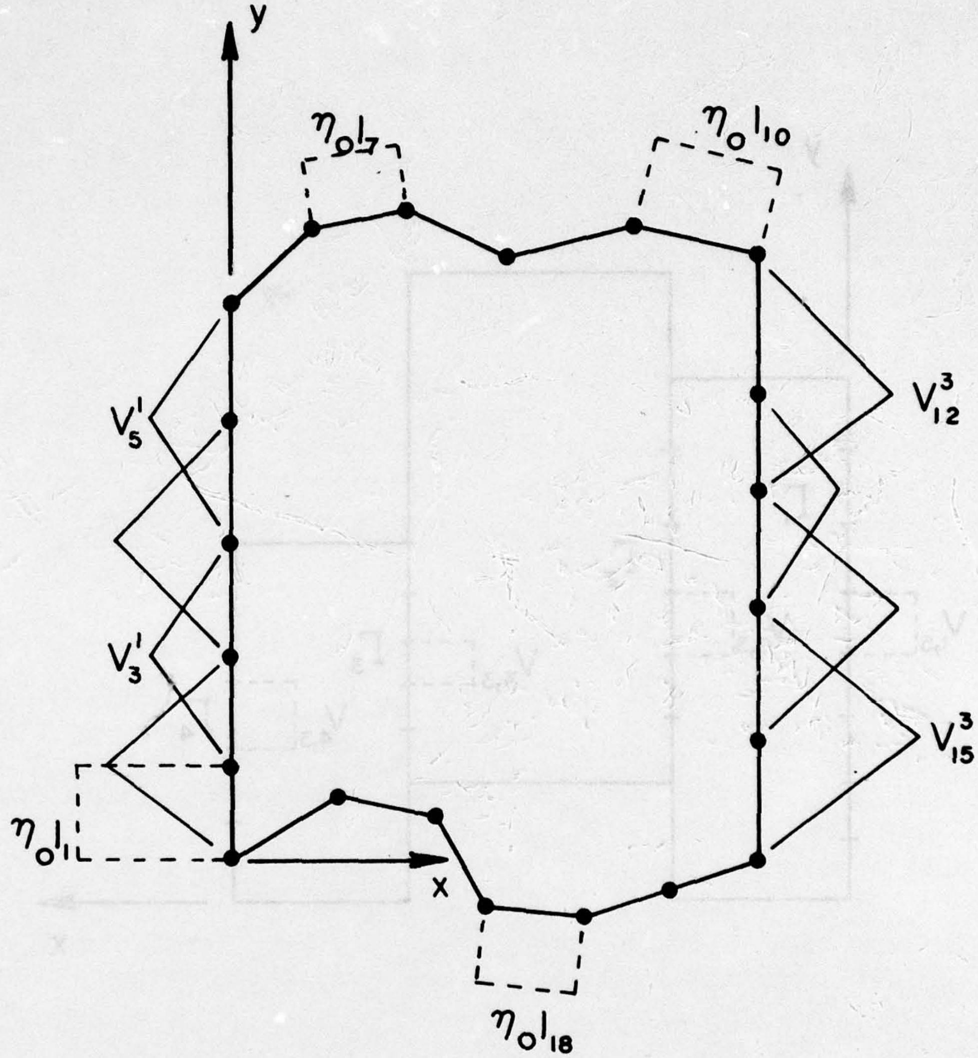

Fig. 7. Placement of some typical elements of TM solution vector on contour C for Program I. Dashed lines represent $\hat{\underline{z}}$ directed expansion functions for $\eta_o\vec{I}$ and solid lines represent $\hat{t}$ directed expansion functions for $\vec{V}^1$ and $\vec{V}^3$, ND = 1,6,11,16,22. The nodes are numbered clockwise from the origin.

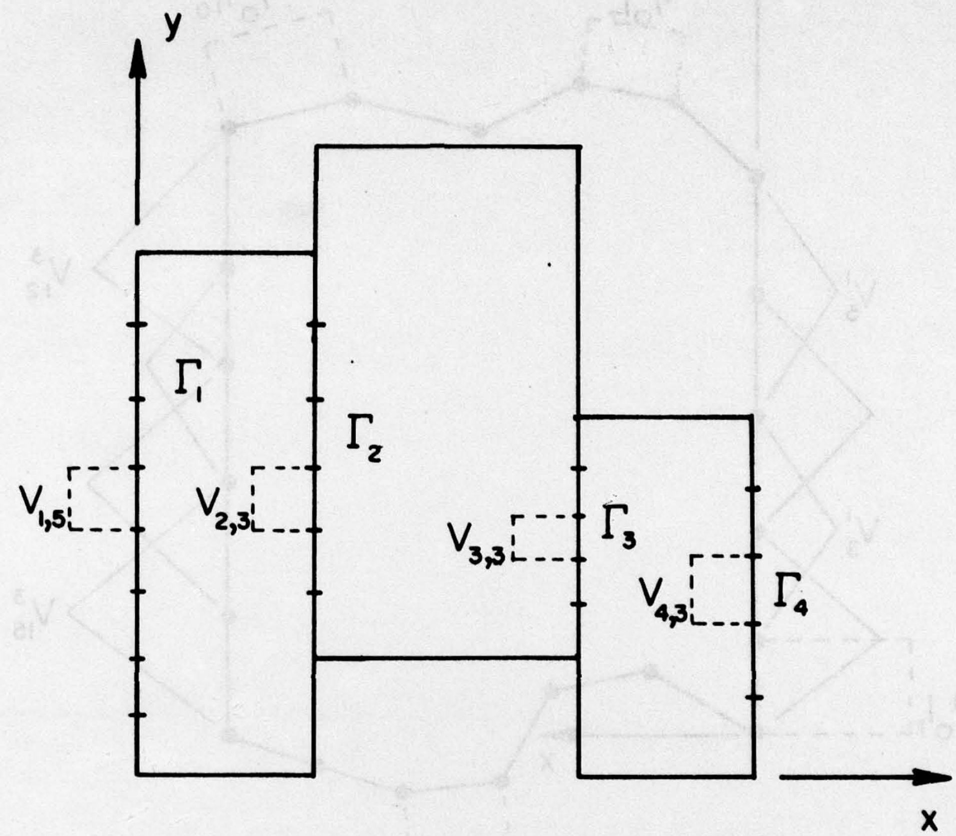

Fig. 8. Placement of some typical elements of TE solution vector on slits Γ_q for Program II. Dashed lines represent $\underline{\hat{z}}$ directed expansion functions for $\vec{V}_1, \ldots, \vec{V}_4$. Numbering is from the bottom to the top of each Γ_q.

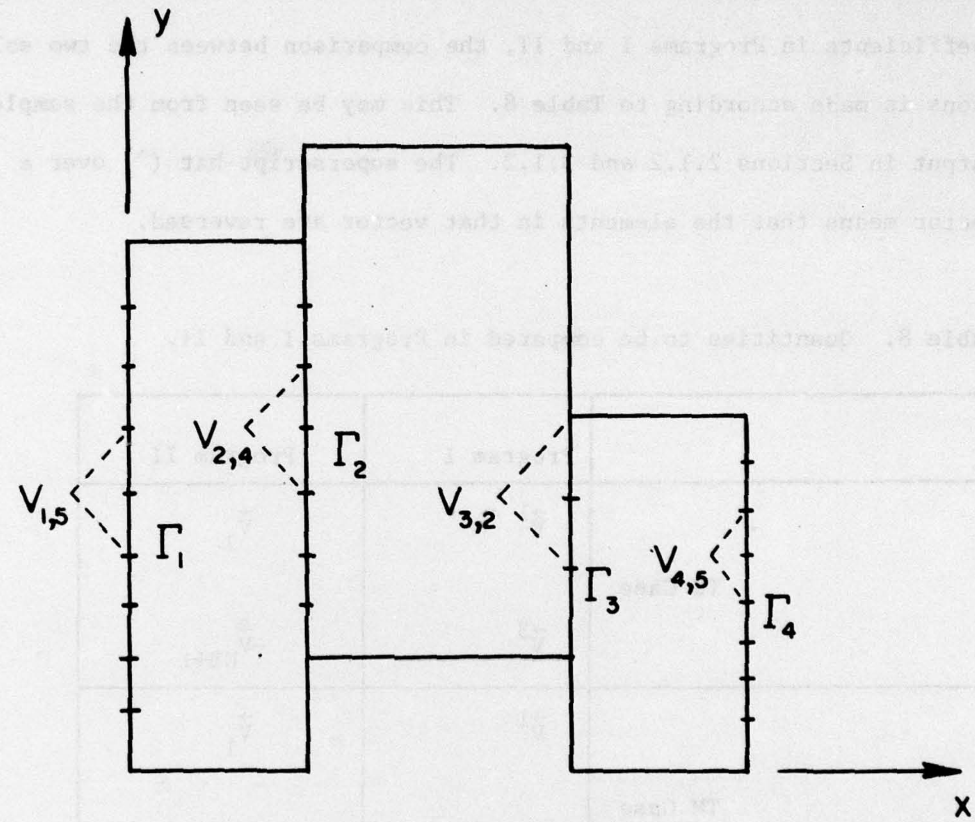

Fig. 9. Placement of some typical elements of TM solution vector on slits Γ_q for Program II. Dashed lines represent $\hat{\underline{y}}$ directed expansion functions for $\vec{V}_1, \ldots, \vec{V}_4$. Numbering is from the bottom to the top of each Γ_q.

These are lines Γ_1 in programs I and II and lines Γ_3 in Program I and Γ_{NB+1} in Program II where the slit cross section has been represented by NB rectangular sub-regions. As a result of the different numbering schemes used for the vectors containing the magnetic current expansion coefficients in Programs I and II, the comparison between the two solutions is made according to Table 8. This may be seen from the sample output in Sections 2.1.2 and 3.1.2. The superscript hat (^) over a vector means that the elements in that vector are reversed.

Table 8. Quantities to be compared in Programs I and II.

	Program I	Program II
TE Case	$\vec{V}^1$	$\vec{V}_1$
	$\vec{V}^3$	$-\hat{\vec{V}}_{NB+1}$
TM Case	$\vec{V}^1$	$\vec{V}_1$
	$\vec{V}^3$	$\hat{\vec{V}}_{NB+1}$

Appendix B

LIST OF VARIABLES USED IN PROGRAM I

The non-modal formulation comprises the bulk of the detail in [1] and hence a list of consistently used Fortran variables, which appear in Program I, is listed here with their counterparts in [1].

Program I variable	Counterpart in [1]
ETA	η_a/η_o
ETB	η_b/η_o
ETC	η_c/η_o
KA	k_a/k_o
KB	k_b/k_o
KC	k_c/k_o
WAVNO	k_o
EPSA	$\varepsilon_a/\varepsilon_o$
EPSB	$\varepsilon_b/\varepsilon_o$
EPSC	$\varepsilon_c/\varepsilon_o$
MUA	μ_a/μ_o
MUB	μ_b/μ_o
MUC	μ_c/μ_o
ULX(m)	$\underline{\hat{t}}_m \cdot \underline{\hat{x}}$
ULY(m)	$\underline{\hat{t}}_m \cdot \underline{\hat{y}}$
ND(i)	N_i
RCX(m)	$x_m + \frac{\Delta C_m}{2} \underline{\hat{t}}_m \cdot \underline{\hat{x}}$
RCY(m)	$y_m + \frac{\Delta C_m}{2} \underline{\hat{t}}_m \cdot \underline{\hat{y}}$

DC(m)	ΔC_m
NGQ	Q of Eq. (A-3)
A(j)	$\frac{A_j^{(Q)}}{2}$ of Eq. (A-3)
T(j)	$\frac{u_j^{(Q)}}{2}$ of Eq. (A-3)
NU(1)	dimension of $\vec{V}^1$
NU(2)	dimension of $\vec{V}^3$
NU(3)	dimension of $\eta_o \vec{I}$
RXK(m)	k_q RCX(m) for q = a,b, or c
RYK(m)	k_q RCY(m) for q = a,b, or c
DCK(m)	k_q DC(m) for q = a,b, or c
U	complex number j
PI	π
PIN(j)	$\eta_o k_o P_{iN}$ of jth excitation
PT(j)	$\eta_o k_o P_{t3}$ of jth excitation

Appendix C

MODAL COMPUTATION OF TANGENTIAL MAGNETIC FIELDS

A comparison is made in Figs. 17-37 of [1] between the tangential magnetic field on the slit cross section from Program I and the tangential magnetic field on the slit cross section computed from the results of Program II. The latter results are obtained from an extra computational effort which is described here. What is needed is the magnetic field inside a closed, perfectly conducting, two-dimensional box due to sheets of magnetic currents. This problem is shown in Fig. 10 where the dimensions of the box are also labelled. The coefficients of the expansion functions for the magnetic currents $\underline{M}_1$ and $\underline{M}_2$, which are different in the two polarizations, are the elements of the vectors $\vec{V}_1$ and $\vec{V}_2$. Each aperture of length W_1 and W_2 is broken up into N_1 and N_2 segments, respectively. The length of these segments are $\Delta_1 = W_1/N_1$ and $\Delta_2 = W_2/N_2$.

For the TE case, a specialization of Eqs. (6-21) and (6-22) to Fig. 10 yields

$$\eta_o H_z = A_o \frac{\cos k_b(x-d)}{\sin k_b d} + B_o \frac{\cos k_b x}{\sin k_b d}$$

$$+ \sum_{p=1}^{\infty} \left[A_p \frac{\cos \alpha_p^1 k_b (x-d)}{\sin \alpha_p^1 k_b d} + B_p \frac{\cos \alpha_p^2 k_b x}{\sin \alpha_p^2 k_b d}\right] \cos \frac{p\pi y}{h}$$

where

$$A_o = \frac{j\eta_o}{\eta_b} \frac{\Delta_1}{h} \left(\sum_{i=1}^{N_1} V_{1,i} \right)$$

$$B_o = \frac{j\eta_o}{\eta_b} \frac{\Delta_2}{h} \left(\sum_{i=1}^{N} V_{2,i} \right)$$

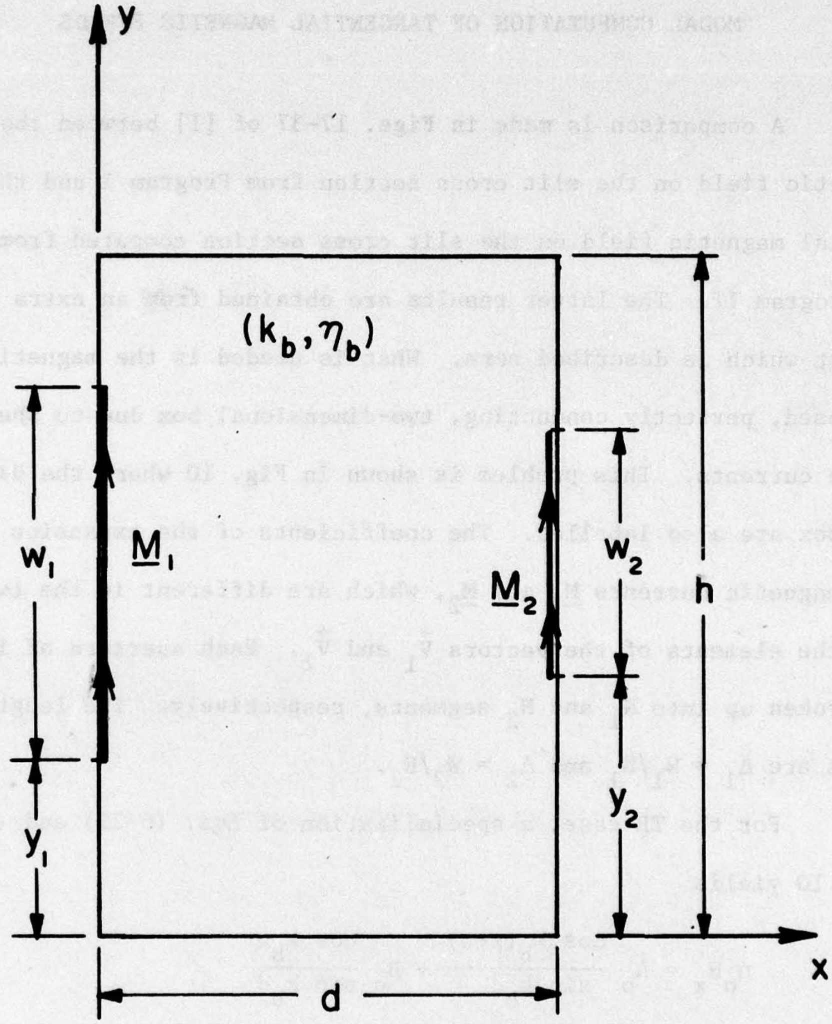

Fig. 10. Magnetic current sheets radiating inside a perfectly conducting two-dimensional box.

$$A_p = \frac{2j\eta_o}{\eta_b} \left(\frac{\sin \frac{p\pi}{2N_1}}{\frac{p\pi}{2N_1}}\right) \frac{\Delta_1}{\alpha_p h} \sum_{i=1}^{N_1} V_{1,i} \cos \frac{p\pi}{h} (y_1 + (i - .5)\Delta_1)$$

$$B_p = \frac{2j\eta_o}{\eta_b} \left(\frac{\sin \frac{p\pi}{2N_1}}{\frac{p\pi}{2N_i}}\right) \frac{\Delta_2}{\alpha_p h} \sum_{i=1}^{N_2} V_{2,i} \cos \frac{p\pi}{h} (y_2 + (i - .5)\Delta_2)$$

and
$$\alpha_p^1 = \sqrt{1 - \left(\frac{p\pi}{k_b w_1}\right)^2}$$

$$\alpha_p^2 = \sqrt{1 - \left(\frac{p\pi}{k_b w_2}\right)^2}$$

This allows the $\hat{\underline{z}}$ directed magnetic field to be computed at any point inside the box of Fig. 10 and when the above is specialized to the walls of the box one may obtain the points labeled "modal J" in Figs. 17-37 of [1]. The computation is rather straightforward and a computer program is not described here for accomplishing this task.

For the TM case, a specialization of Eq. (6-33) to Fig. 10 yields

$$\eta_o H_y = \sum_{p=1}^{\infty} \left[A_p \frac{\cos \alpha_p^1 k_b (x-d)}{\sin \alpha_p^1 k_b d} + B_p \frac{\cos \alpha_p^2 k_b x}{\sin \alpha_p k_b d}\right] \sin \frac{p\pi y}{h}$$

for the y component of magnetic field where

$$A_p = \frac{2j\eta_o}{\eta_b} \alpha_p^1 \frac{\Delta_1}{h} \left[\frac{\sin \left(\frac{p\pi}{2N_1}\right)}{\frac{p\pi}{2N_1}}\right]^2 \sum_{i=1}^{N_1 - 1} V_{1,i} \sin \frac{p\pi}{h} (y_1 + i\Delta_1)$$

$$B_p = \frac{2j\eta_o}{\eta_b} \alpha_p^2 \frac{\Delta_2}{h} \left[\frac{\sin \frac{p\pi}{2N_2}}{\frac{p\pi}{2N_2}}\right]^2 \sum_{i=1}^{N_2 - 1} V_{2,i} \sin \frac{p\pi}{h} (y_2 + i\Delta_2)$$

and α_p^1, α_p^2 are given in the TE case. For the x component of magnetic field, one may easily obtain

$$\eta_o H_x = \sum_{p=1}^{\infty} \left[A_p \frac{\sin \alpha_p^1 k_b (x-d)}{\sin \alpha_p^1 k_b d} + B_p \frac{\sin \alpha_p^2 k_b x}{\sin \alpha_p^2 k_b d}\right] \cos \frac{p\pi y}{h}$$

where

$$A_p = \frac{-2j}{k_b h} \frac{\eta_o}{\eta_b} \frac{\Delta_1}{p\pi h} (2N_1)^2 \sin^2 \frac{p\pi}{2N_1} \sum_{i=1}^{N_1 - 1} V_{1,i} \sin \frac{p\pi}{h} (y_1 + i\Delta_1)$$

$$B_p = \frac{-2j}{k_b h} \frac{\eta_o}{\eta_b} \frac{\Delta_2}{p\pi h} (2N_2)^2 \sin^2 \frac{p\pi}{2N_2} \sum_{i=1}^{N_2 - 1} V_{2,i} \sin \frac{p\pi}{h} (y_2 + i\Delta_2)$$

and α_p^1, α_p^2 are given in the TE case. As pointed out in [1], convergence problems arise when using the formula for $\eta_o H_y$ at Γ_1 or Γ_2.

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